COOPERATIVE DIVERSITY
COMMUNICATIONS OVER BLOCK FADING
CHANNELS

A Thesis Submitted
to the College of Graduate Studies and Research
in Partial Fulfillment of the Requirements
for the Degree of Master of Science
in the Department of Electrical and Computer Engineering
University of Saskatchewan

by
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Acknowledgments

I would like to express my deepest appreciation and gratitude to my supervisor, Professor Ha Nguyen for his support and guidance during my studies. He has been a great source of knowledge and has inspired me with many ideas which are very useful for my research. His patience, encouragement and availability made this thesis possible.

I would like to thank Professor Tho Le-Ngoc of McGill University for his insightful comments and suggestions that he contributed to my research project.

The financial support in the form of a Graduate Scholarship from the Government of Vietnam is gratefully acknowledged. This financial source is important to help me working on my research and finish this thesis.

I would like to say thank you to my labmates as well as all of my friends here for their support and encouragement. Without them, life here would have been more difficult.

My deepest love and gratitude is devoted to all my family members, especially my wife and son, who always support me to overcome the difficulties during my study. To them, I dedicate this thesis.

Finally, I would not forget to say thank to all of my colleagues at Telecommunication Faculty, Posts and Telecommunications Institute of Technology, who put my duties on their shoulders while I am away for study.
Abstract

In a block fading environment, having a poor channel gain over multiple symbol periods is the main reason for the poor error performance of a communications system. Dealing with such a severe effect is a broad, interesting and important research subject. Diversity techniques are commonly used as one of the effective solutions.

Cooperative diversity has recently attracted much interest of researchers in the wireless communications research community. This diversity method was invented to overcome the practical difficulties of providing spatial diversity in the uplink of wireless communications. To be exact, it is impractical to build multiple antennas at the wireless terminals due to the power and/or size limitations. It has been shown that cooperative diversity can help to improve both the system throughput and robustness for the wireless systems.

Motivated by the very encouraging results of cooperative diversity previously reported for both uncoded and coded systems as well as the high demands on wireless broadband transmission, the first part of the thesis investigates bandwidth-efficient coded cooperative communications over block fading channels. The proposed bandwidth-efficient cooperative scheme is shown to improve the error performance of a communications system under adverse block fading conditions as well as to provide a high bandwidth efficiency for wireless broadband access. These advantages of the proposed scheme are thoroughly demonstrated by both simulation and analysis.

In order to further reduce the severe effect of burst errors occurring in block fading channels, a coded cooperative protocol with hybrid automatic retransmission request (H-ARQ) with soft combining, which is a combination of temporal diversity provided by H-ARQ and spatial diversity offered by cooperative diversity, is introduced. A method to effectively construct the cooperative transmitted signal using the H-ARQ transmission protocol is described. The significant benefits of the proposed H-ARQ scheme are also clearly demonstrated in the second part of this thesis.
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<td>Third Generation</td>
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<td>4G</td>
<td>Fourth Generation</td>
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<td>ACK</td>
<td>ACKnowledgement</td>
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<tr>
<td>AMC</td>
<td>Adaptive Modulation and Coding</td>
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<td>ARQ</td>
<td>Automatic Retransmission reQuest</td>
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<tr>
<td>ASK</td>
<td>Amplitude Shift Keying</td>
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<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<td>BEP</td>
<td>Bit Error Probability</td>
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<td>BER</td>
<td>Bit Error Rate</td>
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<td>BICM</td>
<td>Bit Interleaved Coded Modulation</td>
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<td>BICM-ID</td>
<td>Bit Interleaved Coded Modulation with Iterative Decoding</td>
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<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
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<td>CDMA</td>
<td>Code Division Multiple Access</td>
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<tr>
<td>CDMA2000 1xEV-DO</td>
<td>CDMA2000 1x Evolution-Data Optimized</td>
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<tr>
<td>CRC</td>
<td>Cyclic Redundancy Check</td>
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<td>dB</td>
<td>Decibel</td>
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<td>DS-SS</td>
<td>Direct Sequence Spread Spectrum</td>
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<td>FEC</td>
<td>Forward Error Correction Codes</td>
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<td>FER</td>
<td>Frame Error Rate</td>
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<td>FFT</td>
<td>Fast Fourier Transform</td>
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<td>H-ARQ</td>
<td>Hybrid ARQ</td>
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<td>ICI</td>
<td>Inter Channel Interference</td>
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<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
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<tr>
<td>ISI</td>
<td>Inter Symbol Interference</td>
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<td>LLR</td>
<td>Log-likelihood Ratio</td>
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<td>LOS</td>
<td>Line of Sight</td>
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<tr>
<td>NACK</td>
<td>Negative ACKnowledgement</td>
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<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
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<tr>
<td>Abbreviation</td>
<td>Full Form</td>
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<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
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<td>MRC</td>
<td>Maximum Ratio Combining</td>
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<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
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<td>PEP</td>
<td>Pairwise Error Probability</td>
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<td>PN</td>
<td>Pseudonoise</td>
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<tr>
<td>PSD</td>
<td>Power Spectrum Density</td>
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<td>PSK</td>
<td>Phase Shift Keying</td>
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<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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<tr>
<td>QoS</td>
<td>Quality of Service</td>
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<tr>
<td>RCPC</td>
<td>Rate-Compatible Punctured Convolutional Code</td>
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<tr>
<td>RSC</td>
<td>Recursive Systematic Convolutional Code</td>
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<tr>
<td>SC</td>
<td>Systematic Convolutional Code</td>
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<tr>
<td>SISO</td>
<td>Soft-Input Soft-Output</td>
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<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
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<tr>
<td>TC</td>
<td>Turbo Code</td>
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<tr>
<td>TCP/IP</td>
<td>Transmission Control Protocol/Internet Protocol</td>
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<tr>
<td>W-CDMA</td>
<td>Wideband CDMA</td>
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<tr>
<td>WLAN</td>
<td>Wiress Local Area Network</td>
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<tr>
<td>WiMAX</td>
<td>Worldwide Interoperability for Microwave Access</td>
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List of Symbols

\( f(\cdot) \)  
Probability Density Function

\( P(\cdot) \)  
Probability Function

\( Q(x) \)
\[ \frac{1}{2\pi} \int_{x}^{\infty} e^{-t^2/2} dt \]

\( E_b/N_0 \)  
Information Bit Energy to Noise Density Ratio

\( D_s \)  
Doppler Spread

\( L \)  
Frame Length

\( T_c \)  
Coherence Time

\( W_c \)  
Coherence Bandwidth

\( h \)  
Fading Coefficient

\( r \)  
Received Signal

\( r_c \)  
Code Rate

\( s \)  
Modulated Symbol

\( w \)  
Additive White Gaussian Noise

\( \eta \)  
Throughput

\( \rho \)  
Cooperative Level

\( \gamma \)  
Repetition Level

\( \xi \)  
Instantaneous Signal-to-Noise Ratio
1. Introduction

Until a few years before the end of last century, all cellular networks mainly provide voice and low data rate services. The common characteristic of those services is that they require stable network resources such as transmission power, transmission time slot, and a reasonable link reliability. For example, it requires the transmission rate of 13 kbps and bit error rate (BER) level of $10^{-3}$ to offer a voice call over the Global System for Mobile Communications cellular network with a satisfactory quality of service (QoS) [1].

However, with the rapid development and deployment of 3G cellular networks, many new data services were born to provide the end users multimedia applications such as multimedia message services, video telephone, multiplayer games, audio and video streaming, web surfing, content download, and multicast services [2]. These applications require dynamic and huge network resources. For example, the required BER level is of $10^{-5}$ to guarantee the required QoS of data transmission over 3G cellular networks [3].

In order to have a clear idea of the continuously increasing demands on high data rate services, consider the subscriber statistics provided by the CDMA development group during the last 3 years, between September 2003 and September 2006 [4]. As illustrated in Fig. 1.1, from September 2003 to September 2004, the number of users subscribing to the 3G-CDMA2000 1xEV-DO networks for data applications increased by more than 3 fold, from 2.968 million to 10.03 million [4]. For the later two-year period, an approximate two-fold increasing rate is achieved per year. It is predicted that the number of subscribers will continue to increase in the last few years of this
decade.

![Bar chart showing the total number of 3G-CDMA2000 1xEV-DO subscribers over time.](image)

**Figure 1.1** Total number of 3G-CDMA2000 1xEV-DO subscribers.

On the technical side, some signal processing techniques and medium access control protocols have been deployed to meet the requirements of many data applications. They include the use of high-order modulation schemes (QPSK, 8-PSK, 16-QAM), turbo channel coding, hybrid automatic retransmission request (H-ARQ), transmit diversity, scheduling, etc [2,5]. However, the need for even more advanced techniques is still pressing. Especially, this need is great when operators and manufacturers are planning to build wireless infrastructures to provide wireless broadband access with the same quality or better than that of the current fixed networks [6].

Among many promising techniques, which are geared for a long term evolution of 3G networks, the multiple-input multiple-output (MIMO) technique is a primary choice [6]. This technique has attracted much researchers’ attention in recent years under the subject of “space-time coding” [7]. It can be used either to increase the system coverage, capacity or to improve the system performance. The simple and
elegant space-time coding technique, called the Alamouti scheme [8], has been already adopted in the current 3G wireless networks [5, 9]. However, the deployment of this scheme is practically restricted to the downlink communications only. It is realized that there are challenges to practically deploy multiple antennas for the uplink communications. This is because, typically, the spacing between antennas, which depends on the channel characteristic, atmosphere, terrain, and transmission bandwidth, must be on the order of several carrier wavelengths apart to make spatial diversity effective [5, 10].

Fortunately, the challenges of practical spatial diversity deployment in the uplink wireless communications have been relieved since the proposal of a new diversity technique, called user diversity or cooperative diversity [11–21]. The idea of this new spatial diversity technique is that single-antenna users under the serving of the same destination cooperate to transmit information to the destination. As a result, spatial diversity is obtained with single-antenna users. Though this diversity method was proposed just a few years ago, it has already been proposed for the uplink communications of mobile worldwide interoperability for microwave access (Mobile WiMAX) [22], an emerging technology for the wireless broadband access based on the IEEE 802.16 standard.

Motivated by the promising development trend of the current and future wireless networks and the potential advantages of user diversity, this thesis focuses on designing the transmission protocols to improve the data rate and/or the link reliability based on the framework of user diversity. Following is the outline of the research contributions and organization of this thesis.

1.1 Thesis Contribution

In a block fading environment, the fading realization is unchanged over multiple symbol periods [23]. This leads to a very poor error performance of wireless communication systems. In particular, if the channel falls in the deep fading state, it can drive the communication system into out-of-service state, i.e., the communications is
unavailable during the deep fade period. To deal with deep fading, the first contribution of the thesis is a solution to improve the error performance in block fading environment by using cooperative diversity transmission. Furthermore, the proposed scheme is also a viable option for high speed transmission of the current and future wireless networks.

It is observed from the investigation on cooperative transmission that although cooperative diversity significantly improves the system performance, burst errors still happen when both uplink channels fall in the deep fading state. A solution to cope with the burst error phenomenon is the second contribution of this thesis. In this part, the H-ARQ soft combining transmission protocol is proposed for pairwise cooperative communications systems. The aim of the proposed scheme is to obtain as much diversity order as possible by combining both temporal diversity offered by H-ARQ transmission and spatial diversity provided by cooperative transmission to deal with the burst errors. Simulation and analytical results demonstrate that the proposed transmission scheme is able to reduce not only the effect of burst errors caused by block fading, but also the effect of imperfect inter-user channels on end-to-end error performance. The scheme can provide up to 3.5 dB transmit power gain compared to the conventional scheme.

1.2 Thesis Organization

The remaining of this thesis is organized as follows.

Chapter 2 gives an overview of the characteristics of fading channels and some typical techniques to combat adverse fading condition.

Chapter 3 provides the basic concepts of user diversity or cooperative diversity for both uncoded and coded systems as the background for the following chapters. The chapter starts with the introduction of the cooperative concept and an example of how to construct the cooperative transmitted signals for the wireless fading channel. Along with the description of a simple cooperative strategy [11, 12], the optimal
and suboptimal receivers are presented. The throughput is provided to illustrate the significant benefits of the cooperative diversity. The chapter is concluded by introducing a cooperative diversity scheme for coded systems [16], where channel coding such as rate-compatible punctured convolutional codes and turbo codes is used to construct the cooperative transmitted signal.

The first contribution of the thesis is presented in Chapter 4, where a bandwidth-efficient coded cooperative protocol is proposed for a pairwise cooperative system. Descriptions of the encoder and decoder of the proposed scheme are first provided. Then, performance analysis is carried out. Finally, simulations are provided to confirm the analytical derivation and demonstrate the advantages of the proposed scheme.

Chapter 5 is devoted to a coded cooperative protocol with H-ARQ soft combining to cope with the burst error phenomenon caused by block fading. As in Chapter 4, the chapter starts with the description of the encoder and decoder. Then, analysis is performed to verify the validity and benefits of the proposed scheme. Comments and discussions on the performance of the scheme and its potential applications closes the chapter.

Finally, Chapter 6 draws the conclusions and gives suggestions for further studies.
2. Fading Channels and Techniques to Combat Fading

2.1 Fading Channels

In wireless communications, understanding the characteristic of the channel is very useful not only for the link budget design but also the system design such as the transmission protocol, optimal and suboptimal receiver designs. In contrast with wire-line communications, the received signal strength in wireless communications varies with both time and frequency. The variation of the signal strength can be categorized into two types [24]:

**Large-scale variation:** It is well-known that the average value of a radio signal at any point depends on its distance from the transmitter, the carrier frequency, the type of antennas used, antenna heights, atmospheric conditions, etc. It may also vary because of shadowing caused by terrain and clutter such as hills, buildings, and other obstacles [24]. This type of signal variation, which is observable over relatively long distances (a few tens or hundreds of wavelengths of the radio frequency carrier), has a log normal distribution and is termed a large-scale variation.

**Small-scale variation:** As shown in Fig. 2.1, the transmitted signal may arrive at the destination over a number of different paths after being reflected, scattered and diffracted from tall buildings, towers, hills, etc [24]. Because the signal received over each path has a random amplitude and phase, the instantaneous value of the composite signal is found to vary randomly about a local mean. A fade is said to occur when the signal falls below its mean level as shown in Fig. 2.2. The fade is
called a deep fade if the received signal deeply drops below the average level. When
the channel falls in the deep fade, the system performance is strongly degraded.
Therefore, small-scale variation is quite relevant to the transmission protocol and
optimal receiver designs. The research in this thesis focuses on dealing with small-

call
scale variation.

2.2 Channel Modeling

The radio channel can be characterized by the time channel impulse response
$h(\tau, t)$ or by the time-varying channel transfer function $H(f, t)$, which is the Fourier
transform of $h(\tau, t)$. The channel impulse response represents the response of the
channel at time $t$ due to an impulse applied at time $t - \tau$. The radio channel is
assumed to be a wide-sense stationary random process, i.e., the channel has a fading
statistic that remains constant over short periods of time or small spatial distances. In
environments with multipath propagation, the channel impulse response is composed
of a large number of scattered impulses received over $N_p$ different paths [25]. It can

Figure 2.1 A typical wireless channel scenario.
The received signal over a wireless channel.

be represented as:

\[ h(\tau, t) = \sum_{p=0}^{N_p-1} a_p e^{j(2\pi f_{D,p} t + \varphi_p)} \delta(\tau - \tau_p(t)) \] (2.1)

where the function \( \delta(t) \) is defined as

\[ \delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases} \] (2.2)

and

\[ \int_{-\infty}^{+\infty} \delta(t) dt = 1 \] (2.3)

and \( a_p, f_{D,p}, \varphi_p, \) and \( \tau_p(t) \) are the amplitude, the Doppler frequency, the phase, and the propagation delay, respectively, associated with path \( p \).

The Doppler frequency \( f_{D,p} \) depends on the velocity \( v \) of the radio station, the speed of light \( c \), the carrier frequency \( f_c \) and the angle of incidence \( \alpha_p \) of the wave corresponding to path \( p \). It is given by [25]:

\[ f_{D,p} = \frac{v f_c \cos(\alpha_p)}{c} \] (2.4)
For the special case when the transmitter, receiver and propagation environment are all stationary, the attenuation $a_p e^{j(2\pi fD_p t + \varphi_p)}$ and propagation delay $\tau_p(t)$ do not depend on time $t$. Hence the time-variant channel in (2.1) becomes the linear time-invariant channel with an impulse response:

$$h(\tau) = \sum_{p=0}^{N_p-1} a_p e^{j\varphi_p} \delta(\tau - \tau_p) \quad (2.5)$$

For the time-varying impulse response $h(\tau, t)$, the time-varying frequency response $H(f, t)$ is

$$H(f, t) = \int_{-\infty}^{+\infty} h(\tau, t) e^{-j2\pi f \tau} d\tau = \sum_{p=0}^{N_p-1} a_p e^{j2\pi (fD_p t - f\tau_p(t)) + \varphi_p} \quad (2.6)$$

- **Slow fading versus Fast fading**: The distinction between slow and fast fading is related to the coherence time $T_c$ of a radio channel, which measures the period of time over which the fading process is correlated. The coherence time of a radio channel is defined as [23]:

$$T_c \approx \frac{1}{D_s} \quad (2.7)$$

where $D_s = \max_{i,j} |\tau_i' - \tau_j'|$ is the Doppler spread of the channel [24]. The quantities $\tau_i'$ and $\tau_j'$ are the first differentials in time of propagation delays corresponding to paths $i$ and $j$, respectively.

Based on coherence time $T_c$, a fading channel is said to be slow if the symbol period $T_s$ is smaller than the channel’s coherence time $T_c$. This fading situation is also called block fading and the terminology “block fading” is chosen to use throughout this thesis. Otherwise, it is considered to be fast. In block fading, a particular fading realization will affect many successive symbols, which lead to burst errors. Whereas, with fast fading, the fading decorrelates from symbol to symbol [23].

- **Flat fading versus Selective fading**: If all the spectral components of the transmitted signal are affected in the same manner, the fading is said to be
frequency non-selective, or frequency flat [23]. This is the case for narrowband systems in which the transmitted signal bandwidth is much smaller than the coherence bandwidth $W_c$. The coherence bandwidth measures the frequency range over which the fading process is correlated and is defined as the frequency bandwidth over which the correlation function of two samples of the channel response, takes at the same time but at different frequencies, falls below a suitable value. In addition, the coherence bandwidth is related to the maximum delay spread by [23]:

$$W_c \approx \frac{1}{\tau_{\text{max}}}$$

(2.8)

where $\tau_{\text{max}} = \max_{i,j} |\tau_i - \tau_j|$, and $\tau_i$ and $\tau_j$ are the propagation delays corresponding to paths $i$ and $j$, respectively [24].

On the other hand, if the spectral components of the transmitted signal are affected by different amplitude gains and phase shifts, the fading is said to be frequency selective. This kind of fading applies to wideband systems in which the transmitted signal bandwidth is larger than the channel bandwidth $W_c$.

The statistics of the fading process characterize the channel and are of importance for channel model parameter specifications [25]. A simple and often used approach is obtained from the assumption that there is a large number of scatterers in the channel that contribute to the signal at the receiver side. The application of the central limit theorem leads to a complex-valued Gaussian process for the channel impulse response. In the absence of line of sight (LOS) or a dominant component, the process is zero-mean. The magnitude of the channel impulse response, denoted by $h$, is a random variable with a Rayleigh distribution, given by [25, 26]:

$$f(h) = \frac{2h}{\Omega} e^{-\frac{h^2}{\Omega}}$$

(2.9)

where $\Omega = \mathbb{E}\{h^2\}$, $\mathbb{E}(x)$ is the statistic average of the random variable $x$. The phase of the channel impulse response is uniformly distributed over the interval $[0, 2\pi]$ [25].

In the case that the multipath channel contains a LOS or dominant component in addition to the randomly moving scatterers, the channel impulse response can
no longer be modeled as zero-mean. Under the assumption of a complex-valued Gaussian process for the channel impulse response, the magnitude $a$ of the channel impulse response has a Rician distribution given by [25]

$$f(h) = \frac{2h}{\Omega} e^{(-\frac{h^2}{\Omega+\pi})} I_0 \left(2h\sqrt{\frac{K}{\Omega}}\right)$$

(2.10)

The Rician factor $K$ is determined by the ratio of the power of the dominant path to the power of the scattered paths, $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind. The phase of the channel impulse response is uniformly distributed over the interval $[0, 2\pi]$.

The Rayleigh fading channel model is used throughout this thesis to illustrate the advantages of the proposed transmission protocol. It is a popular model for fading channels and leads to a closed-form expressions of bound approximations.

### 2.3 Techniques to Combat Fading

![Figure 2.3](image-url)  

**Figure 2.3** Performance comparison between the AWGN and block fading channels.
Due to the variations in the received signal strength, the overall system performance is severely degraded as illustrated in Fig. 2.3. The results plotted on Fig. 2.3 are obtained based on the binary modulation scheme, i.e., binary phase shift keying (BPSK). The bit error rate (BER) of the system in the presence of the additive white Gaussian noise (AWGN), i.e., the non-fading channel, decays exponentially with the average signal to noise ratio (SNR), where SNR is $E_s/N_0$. Here, $E_s$ is the average symbol energy and $N_0$ is the power spectral density (PSD) of AWGN noise. On the contrary, the BER of the same system over the block fading channel, in which the fading coefficient is constant over the whole symbol period and has a Rayleigh distribution as expressed in Equation (2.9), decays linearly with the SNR. The severe degradation of system performance caused by the small-scale variation is the main issue on which the research community in wireless communications area has dedicated time and effort.

![Figure 2.4](image)

**Figure 2.4** Performance improvement with diversity [24].

A natural solution to improve the error performance is to ensure that the information symbols pass through multiple signal paths, each of which fades independently. In this way, reliable communications is possible as long as one of the paths is strong.
This technique is called *diversity* [24], and it can dramatically improve the system performance over fading channels as illustrated in Fig. 2.4, where $L_d$ denotes the number of the independent transmission paths in the diversity transmission and BPSK modulation is also used to obtain the results. It is observed that the higher the number of diversity paths is, the better the system performance becomes. This is because one information symbol is delivered by multiple independent paths. When one path has a weak signal strength, some other paths may have strong signal strength. As a result, the symbols carried in good paths can be used to save the symbols experiencing the deep fade. This can be mathematically expressed as $P_e \approx (1/\text{SNR})^{L_d}$, where $P_e$ is the symbol error probability [24]. This equation says that given a SNR, the symbol error probability will be exponentially reduced if the number of independent paths $L_d$ increases. The number of independent paths $L_d$ is therefore called the “*diversity order*”. Common diversity techniques include: *frequency diversity, temporal diversity, spatial diversity and cooperative diversity*. They are briefly introduced in the following subsections.

### 2.3.1 Frequency Diversity

In wideband channels, the transmitted signal arrives at the receiver end over multiple symbol periods [24]. As a result, the frequency response of the wideband channel varies with frequency, i.e., the channel is frequency selective fading [23]. In this situation, if multiple versions of the transmitted signal are resolved and combined to recover the original information at the receiver, the system performance will be improved. This technique is called *frequency diversity* [24].

To understand the concept of frequency diversity, consider a simple communication situation that only one symbol, denoted by $s[0]$, is sent at time 0, and no symbols are transmitted after that. According to [24], the sampled output of the time-discrete baseband model of the wideband wireless channel, corresponding to the transmitted symbol $s[0]$, can be written as [24]:

$$r[n] = h_n[n]s[0] + w[n], \; n = 0, 1, 2, \cdots, L_d$$  \hspace{1cm} (2.11)
where $h_n[n]$ denotes the $n$th channel filter tap at time $n$.

If we assume that the channel response has a finite number of taps $L_d$, then the delay replicas of the signal provide $L_d$ branches of diversity in detecting $s[0]$, since the tap gains $h_n[n]$ are assumed to be independent. This diversity is achieved by the ability to resolve the multipaths at the receiver due to the wideband nature of the channel, and is thus called frequency diversity.

A simple communication scheme can be built on the above idea by sending one information symbol every $L_d$ symbol times. The maximal diversity gain of $L_d$ can be achieved, but the problem with this scheme is that it is very wasteful of the bandwidth efficiency. Therefore, in this setting, one can try to transmit symbols more frequently to increase the bandwidth efficiency, but inter-symbol interference (ISI) is an issue. ISI is a phenomenon that the delayed replicas of previous symbol interfere with the current symbol as shown in Fig. 2.5. The problem in frequency diversity is how to deal with ISI while at the same time exploiting the inherent frequency diversity of the channel. In general, there are three common approaches [24]:

![Illustration of inter-symbol interference.](image)

**Figure 2.5** Illustration of inter-symbol interference.

- **Single carrier with equalization**: By using linear and non-linear processing algorithms at the receiver, ISI can be canceled out to some extent. Optimal
maximum likelihood (ML) detection of the transmitted symbols can be implemented with the Viterbi algorithm. However, the complexity of this algorithm grows exponentially with the number of channel taps, and it is typically used only when the number of significant taps is small. Alternatively, linear equalizers attempt to detect the current symbol while linearly suppressing the interference from the other symbols, and they have a lower complexity [24].

- **Direct sequence spread spectrum (DS-SS):** As shown in Fig. 2.6, in the DS-SS system, the information sequence is encoded and modulated by a pseudonoise (PN) sequence to transmit over a bandwidth $W$, which is much larger than the information rate $R$ bits/s. The ratio $W/R$ is sometimes called the *processing gain* of the DS-SS system. Because the symbol rate per user is very low in a spread spectrum system, the ISI is typically negligible and equalization is not required. Moreover, by using the *Rake* receiver, frequency diversity is also obtained. In the cellular situation, multiple spread spectrum users can use the same bandwidth while ideally no inter-user interference exists because each user is assigned one orthogonal spreading sequence. Therefore, when considering the number of bits per bandwidth unit, the bandwidth efficiency of the DS-SS system is high even the bit rate of an individual user is low. In addition to providing the frequency diversity and allowing multiple access, DS-SS systems also have some other advantages such as anti-jamming from intentional interferers and achieving message privacy in the presence of other listeners. In the real world, DS-SS technology has been used in both IS-95 CDMA system (2G) and the current 3G networks (CDMA2000 and W-CDMA) [2,5,9].

- **Multi-carrier systems:** The principle of multi-carrier transmission is to convert a single high-rate data stream onto multiple parallel low-rate sub-streams. Each sub-stream is modulated on a separate sub-carrier as illustrated in Fig. 2.7. Since the symbol rate on each sub-carrier is much less than the initial symbol rate, the effects of delay spread, i.e., ISI, significantly decrease, reducing the complexity of the equalizer.
Figure 2.6 A generic block diagram of direct sequence spread spectrum.

Figure 2.7 An example of multi-carrier system with $N_c = 4$ [25].

Recently, multi-carrier systems based on the orthogonal frequency division multiplexing (OFDM) technique have received great attention. This is because the OFDM technique has the several advanced features. The complexity of an OFDM system is reduced by using the advanced digital signal processing techniques, i.e., inverse fast Fourier transform (IFFT) and fast Fourier transform (FFT) can be used to modulate and demodulate the signals, respectively. The complexity of the receiver is further reduced since ISI and inter channel interference (ICI) can be avoided by using a sufficiently long guard interval. Finally,
the OFDM technique allows different types of signal constellations on each sub-carrier so that the adaptive transmission according to the channel conditions can be carried out [25]. However, an important design goal for multi-carrier transmission schemes based on the OFDM technique for a radio channel is that the channel can be considered as time-invariant during one OFDM symbol and that fading over each sub-channel can be considered as flat. Thus, the OFDM symbol duration should be smaller than the coherence time, $T_c$, of the channel and the sub-carrier spacing should be smaller than the coherence bandwidth $W_c$ of the channel. By fulfilling these conditions, the realization of low-complexity receivers is possible [25].

2.3.2 Temporal Diversity

Temporal diversity is achieved by averaging the fading of the channel over time. Typically, the channel coherence time is of order of tens to hundreds of symbols, and therefore the channel is highly correlated across the consecutive symbols. To ensure that the coded symbols are transmitted with independent or nearly independent fading gains, interleaving of codewords is required. Consider an example in Fig. 2.8 where the fading falls into a deep fade in the third symbol period. If no interleaving is used as illustrated in Fig. 2.8-(a), the codeword $s_3$ will be unlikely recovered correctly because the whole codeword is affected by the deep fade. However, when the interleaving is used as in Fig. 2.8-(b), only a portion of the codeword $s_3$ is in the deep fade and the rest is still in a good transmission condition. As a result, there is a high probability that the codeword $s_3$ can be correctly restored at the receiver end.

2.3.3 Spatial Diversity

Temporal diversity requires interleaving and coding over several coherence time periods. When there is a strict delay constraint and/or the coherence time is large, this scheme may not be possible. In this case, other forms of diversity need to be applied. Spatial diversity, also called antenna diversity, can be obtained by placing
Figure 2.8 Illustration of temporal diversity [24].

As an example to illustrate the potential of this diversity method, consider multiple antennas at the transmitter and/or the receiver. If the antennas are placed sufficiently far apart, the channel gains between different antenna pairs fade more or less independently, and independent signal paths are created. The spatial diversity is classified into three different categories: transmit diversity, receive diversity, and transmit and receive diversity as illustrate in Fig. 2.9.
mit diversity in Fig. 2.9-(a). The simplest way to obtain diversity order is to repeatedly transmit one symbol over $L_d$ transmit antennas and over $L_d$ symbol periods. In one symbol period, only one antenna is turned on and all others are turned off [24]. Although this transmission method offers the diversity order of $L_d$, it has a low bandwidth efficiency. Recently, there has been a lots of research activity in space-time coding, a technique that can provide transmit diversity with a very high bandwidth efficiency. The simplest and most elegant space-time coding scheme is the Alamouti scheme [8]. As demonstrated in [24], Alamouti scheme has 4 dB power gain over the repetition code while bandwidth efficiency is maintained. In addition, the Alamouti scheme can be applied for both real and complex signal constellations. This transmit diversity technique has been adopted in several third-generation standards [2,5,9].

In theory, spatial diversity can be implemented at both transmitter and receiver sides. However, putting multiple antennas at the mobile terminals may be impossible or very expensive due to the life time of batteries and/or size limitation. Fortunately, a new diversity method has been recently proposed to provide spatial diversity, yet

**Figure 2.9** Illustration of spatial diversity.
for the single-antenna users. This diversity method is called cooperative diversity.

2.3.4 Cooperative Diversity

Cooperative diversity is a method in which two or more single-antenna users cooperate with one another to deliver their information to the destination. By using cooperative transmission, the information of a given user can arrive at the destination from multiple transmit antennas while each terminal user has only one antenna. Cooperative diversity has some other distinct features compared to the conventional spatial diversity such as the end-to-end performance depends also on the inter-user channel qualities. Furthermore, each user not only carries information for itself but also for its partners. Therefore, the construction of the transmitted signal for cooperative transmission plays a very important role in order to increase the system throughput and robustness. Because cooperative diversity method is the main research topic in this thesis, the following chapter elaborates this diversity method in greater detail.

2.4 Summary

In this chapter, the background on wireless channels was presented along with statistical models of the fading channels. It was pointed out that variation of the received signal strength due to multipath transmission is the main reason for poor performance of the wireless communication systems. Finally, some well-known and new forms of diversity were presented, including frequency diversity, temporal diversity, spatial diversity, and cooperative diversity.
3. Overview of Cooperative Communications

3.1 Introduction

The concept of cooperative diversity was first introduced in 2003 [11,12]. The work in [11] demonstrated the advantages of cooperative diversity by giving the system description and information theoretic analysis. Furthermore, a simple cooperative transmission method was introduced for the code division multiple access (CDMA) systems along with the performance analysis in [12]. It can be considered that these two papers gave birth to a new research area in wireless communications. This new research area has received a strong interest in the research community [13–15]. The results reported in [11–15] show that the error performance of an uncoded cooperative system is influenced by error propagation due to the imperfect inter-user channels. Due to this problem, coded cooperative diversity has been introduced [16–20] to deal with the error propagation phenomenon. Coded cooperative diversity schemes can avoid the effect of error propagation because each user only relays its partner’s information if the cyclic redundancy check (CRC) result is correct.

In this chapter, uncoded cooperative diversity is elaborated in more detail in Section 3.2, which serves as the background of the cooperation concept. Then, coded cooperative diversity is covered in Section 3.3 as a foundation for the following chapters in this thesis.
### 3.2 Cooperative Diversity for Uncoded Systems

Consider a simple pairwise cooperative diversity scheme where users cooperate in pairs to deliver their information to the same destination as shown in Fig. 3.1. For convenience, the two users are denoted as User 1 and User 2. In pairwise cooperation, one user is the partner of the other user, and vice versa. That is, User 1 is the partner of User 2 and User 2 is the partner of User 1. Furthermore, the channels between users and the destination are called the uplink channels while the channels between users are called inter-user channels.

![Pairwise user cooperative diversity](image)

**Figure 3.1** Pairwise user cooperative diversity.

The channel model of a pairwise cooperative system is presented in Fig. 3.2. Here $I_1$ and $I_2$ denote the information streams of User 1 and User 2, respectively. The transmit symbol sequences $s_1$ and $s_2$ of the two users which are transmitted over the fading channels are not only received by the destination, but also by their partners. The mathematical representation of the channel model can be expressed by the following equations [11]:

\[\text{Equations}\]
where $r_0[n]$, $r_{12}[n]$ and $r_{21}[n]$ are the discrete-time baseband received signals at the destination, User 1 and User 2, respectively, at the time index $n$. The quantities $w_0[n]$, $w_{12}[n]$ and $w_{21}[n]$ are zero-mean complex white Gaussian random variables with variance $N_0/2$. The symbol $h_{ij}$, $i = 1, 2$ and $j = 0, 1, 2$, denotes the channel gain between the source $i$ and the destination $j$. Each fading coefficient $h_{ij}$ is a zero-mean complex Gaussian random variable with variance $\Omega_{ij}$. It is assumed that the destination can track the variations in $h_{10}$ and $h_{20}$, User 1 can track $h_{21}$ and User 2 can track $h_{12}$. This implies that the channel information is available at the receiver. Due to the reciprocity of the radio channel, the fading coefficients $h_{12}$ and $h_{21}$ are assumed to be equal. Finally, for simplicity, the statistical model of the fading is assumed to follow the Rayleigh distribution as discussed in Section 2.2.

Given the above model, the problem is to find the best strategy for both users to construct their transmitted signals, given their own data and the received signal from
their partner, and for the destination to employ the optimal reception scheme so that both users are able to maximize their data rate toward the destination [11]. Note that the transmitted signal should be designed to carry the information not only to the destination but also to the partners, so that cooperation is possible. An example of forming the transmitted signal in cooperative communications is presented in the following subsection.

### 3.2.1 Pairwise Cooperative Diversity with CDMA

Consider a conventional CDMA system in which each user has one spreading code, and its information is modulated by means of binary phase shift keying (BPSK). Assume that the users’ codes are orthogonal and the coherence time of the channel is $L$ symbol periods, i.e., all the fading parameters remain approximately unchanged over $L$ symbols.

**Cooperation strategy for the case of $L = 3$:** Without cooperation, during three consecutive symbol periods, the users would transmit [11]:

$$
\begin{align*}
    s_1 : & \quad a_1 b_1^{(1)} c_1[n], \quad a_1 b_1^{(2)} c_1[n + 1], \quad a_1 b_1^{(3)} c_1[n + 2] \\
    s_2 : & \quad \left( \begin{array}{c}
        a_2 b_2^{(1)} c_2[n] \\
        a_2 b_2^{(2)} c_2[n + 1] \\
        a_2 b_2^{(3)} c_2[n + 2]
    \end{array} \right)
\end{align*}
$$

Period 1 Period 2 Period 3

where $b_j^{(i)}$ is User $j$’s $i$th bit, $c_j$ is User $j$’s code, $n$ is the time index, $a_j = \sqrt{P_j/T_s}$, where $P_j$ is User $j$’s power, and $T_s$ is the symbol period.

Now, assume that the two users decide to cooperate. The cooperation strategy chosen should satisfy some basic criteria. First, for a fair comparison with the non-cooperative case, the total number of codes used by the two users, as well as the modulation scheme, should remain the same. Also, the strategy should not be overly complex. With the above criteria, a cooperation strategy was introduced in [11] to provide a significant increase in throughput over the non-cooperative case. Although it is not necessarily the optimal strategy, the strategy suffices for the purpose of demonstrating the advantages of cooperative diversity. The transmit symbols con-
constructed for the cooperation strategy in [11, 12] are as follows:

\[ s_1 : \quad a_{11}b_1^{(1)}c_1[n], \quad a_{12}b_1^{(2)}c_1[n + 1], \quad a_{13}\hat{b}_1^{(2)}c_1[n + 2] + a_{14}\hat{b}_2^{(2)}c_2[n + 2] \] (3.6)

\[ s_2 : \quad a_{21}b_2^{(1)}c_2[n], \quad a_{22}b_2^{(2)}c_2[n + 1], \quad a_{23}\hat{b}_1^{(2)}c_1[n + 2] + a_{24}b_2^{(2)}c[n + 2] \] (3.7)

where \( \hat{b}_j^{(i)} \) is the partner’s estimate of User \( j \)’s \( i \)th bit. The parameters \( \{a_{ij}\} \) control how much power is allocated to a user’s own bits versus the bits of the partner, while maintaining an average power constraint of \( P_j \) for User \( j \) over \( L \) symbol periods.

As shown in Equations (3.6) and (3.7), the users send their own information to the destination in the first period. On the other hand, Period 2 is used to send data not only to the destination, but also to each user’s partner. After this data is estimated by each user’s partner, it is used to construct a cooperative signal that is sent to the destination during Period 3. This is accomplished by each user utilizing both users’ codes (\( c_1[n] \) and \( c_2[n] \)). This is done in such a way as to enable the two partners to send cooperative signals while keeping the total number of codes used by two users constant.

It is noticed that Period 3 is used in order to resend, in some sense, the information originally sent during Period 2. This implies that the users only send two new bits per three symbol periods, whereas they could be sending three new bits per three symbol periods if they were not cooperating. This seems counterproductive, but, under certain channel conditions, “wasting” a few symbol periods for cooperation may be justified\(^1\). This is because the performance criterion is the throughput, the number of bits successfully received per transmission, rather than the number of transmitted bits per symbol.

**Cooperation strategy for an arbitrary coherence time \( L \):** Equations (3.6) and (3.7) refer to the cooperation strategy for the special case of \( L = 3 \). The generalization to an arbitrary value of \( L \) is as follows [11]. Over \( L \) symbol periods, each

\(^1\)It is stated in [11] that: “It may be better to receive 1 very high signal to noise ratio bit per symbol period than to receive 10 very low SNR bits per symbol period.”
of the two partners uses $2L_c$ of the periods for cooperative information and the remaining $L_n = L - 2L_c$ periods for sending non-cooperative information, where $L_c$ is some integer between 0 and $L/2$. When $L_c = 0$, the two users are not cooperating at all. When $L_c = L/2$, the two users are fully cooperating, that is, cooperating occurs during all symbol periods. In general, the value of $L_c$ is not necessarily constant over all time. It can be varied according to the channel conditions.

The cooperation scheme described above for a given $L$ and $L_c$ is as follows [11]:

$$
s_1 = \begin{cases} 
a_{11}b_1^{(i)}c_1[i], & i = 1, 2, \cdots, L_n 
a_{12}b_1^{((L_n+1+i)/2)}c_1[i], & i = L_n + 1, L_n + 3, \cdots, L - 1 
a_{13}b_1^{((L_n+i)/2)}c_1[i] + a_{14}\hat{b}_2^{((L_n+i)/2)}c_2[i], & i = L_n + 2, L_n + 4, \cdots, L 
\end{cases} \quad (3.8)
$$

$$
s_2 = \begin{cases} 
a_{21}b_2^{(i)}c_2[i], & i = 1, 2, \cdots, L_n 
a_{22}b_2^{((L_n+1+i)/2)}c_2[i], & i = L_n + 1, L_n + 3, \cdots, L - 1 
a_{23}b_1^{((L_n+i)/2)}c_1[i] + a_{24}\hat{b}_2^{((L_n+i)/2)}c_2[i], & i = L_n + 2, L_n + 4, \cdots, L 
\end{cases} \quad (3.9)
$$

where $L_n = L - 2L_c$, and $a_{ij}$ are chosen to satisfy the following power constraints [11]:

$$
\frac{1}{L} \left[ L_n a_{11}^2 + L_c (a_{12}^2 + a_{13}^2 + a_{14}^2) \right] = P_1 
\frac{1}{L} \left[ L_n a_{21}^2 + L_c (a_{22}^2 + a_{23}^2 + a_{24}^2) \right] = P_2 
$$

(3.10)

3.2.2 Performance Analysis of Cooperative Diversity with CDMA

Consider the cooperation strategy for the conventional CDMA system described in Subsection 3.2.1. Assume that the received signals are chip-matched filtered at the receivers. Therefore, all signals can be written as length-$N_c$ vectors of chip-matched filter outputs, where $N_c$ is the CDMA spreading gain. Furthermore, assume that the spreading codes, $c_i[n]$, are orthogonal. This implies that no inter-user interference exists at the receiver end (the destination). Finally, for convenience, the performance analysis is derived for User 1 and all irrelevant subscripts and superscripts are removed. User 2’s error probability can be obtained analogously due to the symmetry.
Error probability for non-cooperative period: During the $L - 2L_c$ non-cooperative periods, each user sends only its own data, which is received and detected by the destination only. The signal transmitted by User 1 is $s_1 = a_{11}b_1c_1$, and is received at the destination according to

$$r_0 = h_{10}s_1 + h_{20}s_2 + w_0$$  \hspace{1cm} (3.11)

Due to the orthogonality of the spreading codes, the estimate of User 1’s bit during these periods is given by

$$\hat{b}_1 = \text{sign}\left(\frac{1}{N_c}c_1^Tr_0\right) = \text{sign}(h_{10}a_{11}b_1 + w'_0) \hspace{1cm} (3.12)$$

where $w'_0 \sim \mathcal{N}(0, \sigma_0^2/N_c)$, and $\sigma_0^2 = N_0/2T_c$, $T_c$ is the chip period, and $N_0/2$ is power spectral density (PSD) of the AWGN $w_0(t)$. As a result, the bit error probability is given by [11]:

$$P_{e_1} = Q\left(h_{10}a_{11}\frac{\sqrt{N_c}}{\sigma_0}\right) \hspace{1cm} (3.13)$$

where $Q$-function is defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}dt$.

Error probability for cooperative periods and suboptimal receiver: During the $2L_c$ cooperative periods, there is a distinction between “odd” and “even” periods. During the “odd” periods, each user sends only its own data, which is received and detected by the destination and its partner as well. The signal transmitted by User 1 is $s_1 = a_{12}b_1c_1$. It is received by the partner according to

$$r_{12} = h_{12}s_1 + w_{12} \hspace{1cm} (3.14)$$

and by the destination according to

$$r_{0}^{\text{odd}} = h_{10}s_1 + h_{20}s_2 + w_{0}^{\text{odd}} \hspace{1cm} (3.15)$$

The partner uses the received signal $r_{12}$ in order to form a hard estimate of $b_1$, whereas the destination uses the received signal $r_{0}^{\text{odd}}$ in order to form a soft decision statistic.

The partner’s hard estimate of $b_1$ is given by

$$\hat{b}_1 = \text{sign}\left(\frac{1}{N_c}c_1^Tr_0\right)$$  \hspace{1cm} (3.16)
and the probability of bit error is

\[ P_{e12} = Q \left( h_{12}a_{12} \frac{\sqrt{N_c}}{\sigma_1} \right) \]  

(3.17)

where \( \sigma_1^2 = N_1/2T_c \) and \( N_1/2 \) is the PSD of \( w_{12}(t) \). The destination forms a soft decision statistic by calculating

\[ r_{odd} = \frac{1}{N_c} c_1^T r_{0}^{odd} \]  

(3.18)

The value of \( r_{odd} \) is combined with the information obtained from the following “even” period. During the “even” periods, the two users send the cooperative signals to the destination, based on what each user estimates for his/her partner’s bit from the previous “odd” period. The transmitted signals of the two partners are:

\[ s_1 = a_{13}b_1 c_1 + a_{14}\hat{b}_2 c_2 \]  

(3.19)

\[ s_2 = a_{23}\hat{b}_1 c_1 + a_{24}b_2 c_2 \]  

(3.20)

The destination receives these signals according to

\[ r_{0}^{even} = h_{10}s_1 + h_{20}s_2 + w_0^{even} \]  

(3.21)

and therefore a soft decision statistic is obtained by calculating

\[ r_{even} = \frac{1}{N_c} c_1^T r_{0}^{even} \]  

(3.22)

The destination’s combined decision statistics for User 1 are therefore given by

\[
\begin{align*}
    r_{odd} &= h_{10}a_{12}b_1 + w_0^{odd} \\
    r_{even} &= h_{10}a_{13}\hat{b}_1 + h_{20}a_{23}\hat{b}_1 + w_0^{even}
\end{align*}
\]  

(3.23)

where \( \hat{b}_1 \) is User 2’s estimate of \( b_1 \), with an error probability given by (3.17). It is also noted that \( w_0^{odd} \) and \( w_0^{even} \) are statistically independent and both distributed according to \( \mathcal{N}(0, \sigma_0^2/N_c) \). The detection rule of the optimal receiver, which is the maximum \( a \) posteriori (MAP) probability detector, is derived in [12] as follows:

\[
(1 - P_{e12})A^{-1}e^{y_1^T r} + P_{e12}A e^{\hat{y}_1^T r} \overset{b_1=1}{\geq} (1 - P_{e12})A^{-1}e^{-y_1^T r} + P_{e12}A e^{-\hat{y}_1^T r}
\]  

(3.24)
where \( r = [y_{\text{odd}}, y_{\text{even}}]^T \sqrt{N_c}/\sigma_0 \), \( \mathbf{v}_1 = [h_{10}a_{12}, (h_{10}a_{12} + h_{20}a_{23})]^T \sqrt{N_c}/\sigma_0 \), \( \mathbf{v}_2 = [h_{10}a_{12}, (h_{10}a_{12} - h_{20}a_{23})]^T \sqrt{N_c}/\sigma_0 \), and \( A = e^{(h_{10}h_{20}a_{13}a_{23}N_c/\sigma_0^2)} \).

Unfortunately, this detector is not only complex, but also does not yield a closed-form expression for the resulting bit error probability. Therefore, the analysis of such receiver can be done through computer simulations. To overcome those difficulties, a suboptimal receiver is introduced in [11,12] and it makes the decision as follows:

\[
\hat{b}_1 = \text{sign}([h_{10}a_{12}, \lambda(h_{10}a_{13} + h_{20}a_{23})]r)
\]

where \( \lambda \in [0,1] \) is a measure of the destination’s confidence in the bits estimated by the partner. Specifically, it can be shown that when the destination believes that the inter-user channel is perfect, i.e., \( P_{e_{12}} = 0 \), then the optimal detector in (3.24) collapses to the detector in (3.25) with \( \lambda = 1 \) since this corresponds to the maximal-ratio combining (MRC) detector. As the inter-user channel becomes more unreliable, i.e., \( P_{e_{12}} \) increases, although there is no equivalence between the optimal and suboptimal detectors, the value of the best \( \lambda \) in (3.25) decreases toward zero.

Then, the detector in (3.25) is a modified MRC, where the branch with the partner’s uncertain bit estimates is weighted less than the branch with the bits coming directly from the desired user. This detector is therefore referred to as the \( \lambda \)-MRC. It is pointed out in [12] that the \( \lambda \)-MRC has the following advantages

1. Under most channel conditions, the performance of the \( \lambda \)-MRC approaches that of the optimal detector in (3.24) if the parameter \( \lambda \) is chosen appropriately. This fact is illustrated in Fig. 3.3 for various values of the inter-user channel quality \( P_{e_{12}} \), where \( \xi_1 = h_{10}a_{12}\sqrt{N_c}/\sigma_0 \), and \( \xi_2 = h_{10}a_{13}\sqrt{N_c}/\sigma_0 \) are the instantaneous signal to noise ratio at the receiver end for the non-cooperative bit and the cooperative bit, respectively. Similarly, \( \xi_3 = h_{20}a_{23}\sqrt{N_c}/\sigma_0 \) is the instantaneous signal to noise ratio of the estimated bit which is delivered by the partner.

2. The \( \lambda \)-MRC is very simple and computationally undemanding.

3. It has a closed-form expression for the bit error probability, thus enabling simulation-free system analysis. In particular, independent of how the value
of $\lambda$ is set, the bit error probability for the $\lambda$-MRC is given by [12]:

$$P_{e_2} = (1 - P_{e_1})Q\left(\frac{\mathbf{v}_\lambda^T\mathbf{v}_1}{\sqrt{\mathbf{v}_\lambda^T\mathbf{v}_\lambda}}\right) + P_{e_1}Q\left(\frac{\mathbf{v}_\lambda^T\mathbf{v}_2}{\sqrt{\mathbf{v}_\lambda^T\mathbf{v}_\lambda}}\right)$$ (3.26)

where $\mathbf{v}_\lambda = [h_{10}a_{12}, \lambda(h_{10}a_{13} + h_{20}a_{23})]^T$, $\mathbf{v}_1 = [h_{10}a_{12}, h_{10}a_{13} + h_{20}a_{23}]^T\sqrt{N_c}/\sigma_0$, and $\mathbf{v}_2 = [h_{10}a_{12}, h_{10}a_{13} - h_{20}a_{23}]^T\sqrt{N_c}/\sigma_0$.

![Figure 3.3](image-url)  
**Figure 3.3**  Performance comparison of the $\lambda$-MRC and the optimal detectors [12].

**System throughput:** According to the error analysis in the previous section, for every $L$ symbol periods, the destination receives $L - 2L_c$ bits with a probability of bit error equal to $P_{e_1}$, given in (3.13), and $L_c$ bits with a probability of bit error equal to $P_{e_2}$, given in (3.26). The system throughput is defined in [12] as: “The capacity of a binary symmetric channel with crossover probability equal to the probability of error calculated for a particular scheme”. Based on the error probabilities for both cooperative and non-cooperative periods, the resulting throughput for User 1, i.e., the maximum data rate at which User 1 can transmit reliably using sufficiently long error-correcting codes, is given by [12]:

$$\eta_1(L_c, \{a_{ij}\}, \{h_{ij}\}) = \frac{1}{L}[L_n(1 - H(P_{e_1})) + L_c(1 - H(P_{e_2}))]$$ (3.27)
where \( H(p) \) is the entropy of a Bernoulli random variable with parameter \( p \) [27].

This expression, based on the definitions of \( P_{e1} \) and \( P_{e2} \) above, holds for a particular value for \( L_c \), a particular power allocation, as defined by the \( \{a_{ij}\} \), and a particular set of fading coefficients, as defined by \( \{h_{ij}\} \). Since \( \{h_{ij}\} \) are random variables, the throughput for a given value of \( L_c \) and a given power allocation \( \{a_{ij}\} \) becomes [12]

\[
\eta_1(L_c, \{a_{ij}\}) = \mathbb{E}_{h_{ij}}[\eta_1(L_c, \{a_{ij}\}, \{h_{ij}\})]
\] (3.28)

where \( E(x) \) is the expectation operation. An analogous expression holds for User 2’s throughput. Therefore, given \( L_c \) and a set of \( \{a_{ij}\} \) that remain constant over all realizations of \( \{h_{ij}\} \), the two users achieve the rate pair \((\eta_1(L_c, \{a_{ij}\}), \eta_2(L_c, \{a_{ij}\}))\). By time sharing between different values of \( L_c \) and different sets of \( a_{ij} \), while maintaining the power constraints given in (3.10), the two users are able to attain an achievable rate region that is the convex hull of the set of all \((\eta_1(L_c, \{a_{ij}\}), \eta_2(L_c, \{a_{ij}\}))\) pairs.

\(^2\)The function \( H(p) \) is defined as: \( H(p) = -p \log(p) - (1 - p) \log(1 - p) \).
Fig. 3.4 sketches the achievable throughput for both users (reproduced from [12]) who participate in the cooperation process in the convectional CDMA implementation. More specifically, this results correspond to the scenario that both uplink channels experience the same fading statistics. The coherence period is $L = 8$, and full cooperation is considered (i.e., $L_c = 4$). There is an improvement in capacity for both users in the cooperative system. The achievable throughput for the scenario that the two uplink channels have a dissimilarity of fading statistics is also presented in [12], which shows that the achievable throughput is improved if the cooperative level increases.

### 3.3 Cooperative Diversity for Coded Systems

As indicated in [13–15], the cooperation strategies for uncoded systems, such as the one described in Section 3.2, experience error propagation when the inter-user channels are imperfect. This error propagation degrades the system performance. Therefore, it is necessary to have solutions to combat that negative effect of cooperative communications.

A simple solution is that the information about the inter-user channel quality is sent between a given user and its partner. However, this solution increases the complexity of the terminal devices. In [16–19], cooperation strategies for coded systems have been investigated. To avoid error propagation existing in uncoded systems, a cyclic redundancy check (CRC) code is used to monitor the quality of inter-user channels. Whenever the inter-user channel quality is low (i.e., an user fails to detect its partner’s information), the system automatically switches from the cooperative mode to a non-cooperative mode. As a result, error propagation is eliminated. Although CRC is used for the purpose of avoiding error propagation, it does not change the bandwidth efficiency compared to the non-cooperative coded systems. This is because CRC is always used in the coded systems as a frame quality indicator.

The concept of pairwise coded cooperation is illustrated in Fig. 3.5, where the data of each user is segmented into blocks that are augmented with an CRC code.
Figure 3.5  Illustration of coded cooperative communications [16].

There is a total of $K$ bits per block (including the CRC bits). Each block is then encoded with an error correcting code so that, for an overall code rate of $r_c$, there are $N = K/r_c$ coded bits per block. The two users cooperate by dividing the transmission of their $N$-bit codewords into two successive time segments, or frames. In the first frame, each user transmits a codeword of $N_1$ bits. Each user receives and decodes his partner’s first frame. If the user successfully decodes the partner’s $N_1$-bit codeword, the user computes and transmits $N_2$ additional parity bits for the partner’s data in the second frame, where $N_1 + N_2 = N$. For example, one can use the puncturing technique to obtain a codeword of length $N_1$ bits from the original $N$-bit codeword. If the puncture code is used in the first frame, a user will produce the $N_2$ punctured bits ($N_2 = N - N_1$) based on its partner’s Frame 1. The $N_2$ punctured bits are delivered to the destination in Frame 2 by the partner.

Whenever a user is unable to successfully decode his partner’s message, the user will revert to a non-cooperative mode by calculating and transmitting $N_2$ parity bits for his own message. If a user successfully decodes the partner but not vice versa, both users will transmit the partner’s bits in the second frame. These bits are optimally
combined at the destination prior to decoding. The destination needs to know whose
bits each user is transmitting in the second frame. A simple solution is that the
destination can simply decode according to each of possibilities in succession until
successful decoding. It is quite obvious that the above cooperation strategy for coded
systems maintains the overall system performance and the rate at the cost of some
added complexity at the destination.

In coded cooperation, each user always transmits a total of $N$ bits per source
block over the two frames, and the users only transmit in their own multiple access
channels. The cooperative level is defined as $\rho = N_2/N$, which is the percentage of
the total bits per each source block that the user transmits for his partner. In the next
chapter, the role of cooperative level on the system performance will be considered.

**An example of coded cooperation with turbo code:** Various channel cod-
ing methods can be used within the coded cooperative framework. For example,
the overall code may be a block or convolutional code or the combination of both.
The partitioning of the coded bits for two frames may be obtained by using the
rate-compatible punctured convolutional (RCPC) codes. However, since coded coop-
eration involves two code components, turbo codes (TC) are a natural fit.

The implementation of coded cooperation using turbo codes is shown in Fig. 3.6.
Turbo codes employ two constituent recursive systematic convolutional (RSC) codes
with an interleaver [28], denoted by $\Pi$, which is the same for both the users and the
destination. The codeword for the first frame is obtained by using the first RSC code.
Upon successful decoding of the partner data, which is based on the CRC check result,
the user interleaves the source bits over the $K$-bit block and transmits the parity bits
corresponding to the second RSC code [16]. In turbo coded cooperation, each user
transmits his partner’s new parity bits in Frame 2 without repeating any old bits as
in the uncoded systems. If the first frame of the partner is not successfully decoded,
the user will interleave, encode, and transmit the second set of parity bits for its own.
3.4 Summary

In this chapter, the concepts of cooperative diversity were presented for both uncoded and coded systems. By providing a simple cooperation strategy for uncoded CDMA system in Subsection 3.2, the implementation of cooperative communications was illustrated. Furthermore, the optimal and suboptimal detection solutions were also described. The investigation of the system throughput shows that cooperative diversity plays an important role in increasing the user capacity. The cooperation strategy for coded systems was elaborated in Section 3.3. It is shown that error propagation, which exists in uncoded systems, can be eliminated. Turbo coded cooperation was chosen to illustrate how the transmitted signals are constructed in the coded cooperative scenario. The framework of turbo coded cooperation shall be adopted in Chapter 4 to investigate our proposed cooperative protocol in block fading environments, which is one of the contributions of the thesis.

Figure 3.6 Turbo encoding in coded cooperative communications [16].
4. Bandwidth-Efficient Coded Cooperative Communications over Block Fading Channels

4.1 Introduction

As discussed in Chapter 2, the main problem of wireless communication is the variation of the received signal strength or fading, which is due to multi-path transmission and movement of wireless terminals. The overall system performance is especially seriously degraded in block fading as illustrated in Fig. 2.3 because one fading realization is almost unchanged over multiple symbol periods [23]. Therefore, if the channel falls into a deep fade, likely the system will be in outage or the quality of service will not be guaranteed.

Recently, feedback from customers using data applications of 3G wireless networks indicate that high speed transmission is necessary not only for the downlink but also for the uplink communications to guarantee the QoS of some typical data applications which are based on the transmission control protocol/internet protocol (TCP/IP) [29, 30]. Motivated from such demand of high speed transmission for the uplink of the current and future wireless networks, this chapter considers applying high-order modulation schemes for the coded cooperative diversity technique, which is designed mainly for the uplink communications [11–20,31] and has been previously investigated with the simple modulation scheme of BPSK [16].

Moreover, a bandwidth-efficient cooperative transmission protocol is investigated for the coded systems. This is because, as discussed in Chapter 3, coded systems have the ability to alleviate error propagation caused by the imperfect inter-user
channel. Bandwidth-efficient coded communications systems can be implemented by using either the trellis coded modulation (TCM) [28] or bit interleaved coded modulation (BICM) scheme [32]. However, BICM is selected in this chapter because this scheme is more flexible to implement. For example, the selection of channel coding can be made independent of the modulation scheme, and vice versa [32].

In addition, as discussed in Chapter 3, the structure of turbo codes [28] is a natural fit to the coded cooperative systems. Therefore, turbo codes are also used in the proposed bandwidth-efficient cooperative scheme. It is noted that the systematic bits of turbo codes are always transmitted to the destination only one time in the previously proposed protocol [16]. Here, when applying turbo codes to the proposed cooperative diversity transmission in a block fading environment, the systematic bits are also repeated. The aim of repeating the systematic bits is to increase the number of information bits enjoying diversity order 2. However, it is also well-known that the bandwidth-efficiency of a system is reduced if repetition transmission is carried out. To maintain the same bandwidth efficiency as that of the schemes that do not repeat systematic bits, higher-order modulation schemes must be used in the proposed system.

Typically, if a higher-order modulation is used, error performance becomes poorer due to the reduction of the minimum distance of the higher-order constellation (when the average transmitted power is maintained). However, it will be demonstrated in Section 4.4 that if the repetition level is appropriately selected, the proposed scheme can improve the error performance, and approximately 2 dB gain can be obtained.

Furthermore, since the current wireless networks (3G) are mainly designed for “bursty” data applications [2, 5, 9], link adaptive techniques are preferable to power control techniques (as in the legacy networks that mainly serve voice services). Here a new link adaptive technique for the proposed coded cooperative system and its potential throughput gain are also discussed.

The remaining of the chapter is organized as follows. The proposed coded coop-
ervative scheme for high bandwidth efficiency is described in Section 4.2. Performance analysis is provided in Section 4.3. Section 4.4 presents analytical and simulation results for different $M$-PSK and $M$-QAM systems in various block fading scenarios. Also included in this section is a discussion of a new link adaptation approach and its advantages. Section 4.5 gives conclusions.

4.2 System Description

4.2.1 Encoder

Consider a pairwise cooperative system as illustrated in Fig. 3.1 in which two users, called User 1 and User 2, cooperate with each other to send their information to the third node, called the destination. In this case, User 1 is the partner of User 2 and vice versa. Fig. 4.1 shows the block diagram of the encoding process in the proposed bandwidth-efficient coded cooperative system for User 1 (similar for User 2). The coded cooperation happens over two frames as follows:

- **Frame 1**: An information block of $K$ bits (including the CRC bits) is fed to the first systematic convolutional encoder (SC1) to produce the first parity-bit sequence. Both the systematic-bit and the first parity-bit sequences are interleaved by the interleaver $\Pi_1$. Then, the interleaved bits are mapped to the $M$-ary signal constellation to produce the symbol sequence for Frame 1. Over Frame 1, User 1 also receives the transmitted signal of User 2’s Frame 1. This implies that User 1 can operate in a full-duplex mode [11–14,16–18]. A similar process applies to User 2. The encoding carried out in Frame 1 is generally known as bit-interleaved coded modulation (BICM) [32].

- **Frame 2**: Based on the partner’s received signal in Frame 1, each user implements BICM decoding to restore the original information for its partner. Whether or not a user successfully decodes the information for its partner is indicated by the CRC result. If CRC is correct, the decoded information is first interleaved by the interleaver $\Pi$, then applied to the second systematic convolu-
Figure 4.1  Encoder in the proposed coded cooperative communications.

ational encoder (SC2) to produce the second parity-bit sequence. Different from the previously proposed coded cooperative system with BPSK [16] where no information bits are repeated by the partner, here the systematic bits together with the parity bits produced by SC2 are combined and mapped to the $M$-ary signal constellation and sent to the destination in Frame 2. Moreover, to provide a flexible spectral efficiency, the portion of the systematic bits repeated in Frame 2 is controlled by a puncturer as seen in Fig. 4.1.

For convenience, define the parameter $\gamma$, called the *repetition level*, that represents the percentage of the systematic bits repeated by the partner as follows:

$$\gamma = \frac{\text{Number of systematic bits repeated}}{\text{Total number of systematic bits}} \quad (4.1)$$

By definition, the range of $\gamma$ is from 0% to 100%. It can be seen that a higher repetition level indicates a larger number of systematic bits enjoying a diversity order of 2 at the expense of lower transmission efficiency. In order to maintain a given bandwidth efficiency, higher-order modulation schemes must be used. Consider a system using an $M$-ary modulation scheme with identical SC1 and SC2 of rate $r_c$,.
and a repetition level \( \gamma \). It can be derived that the cooperative level is

\[
\rho = \frac{1 - (1 - \gamma)r_c}{2 - (1 - \gamma)r_c}
\]

(4.2)

and the overall system bandwidth efficiency is

\[
\eta_{BW} = \frac{r_c \log_2 M}{2 - (1 - \gamma)r_c} \text{ (bits/sec/Hz)}
\]

(4.3)

For example, consider the specific case of coded cooperation with block length \( K = 132 \) information bits and identical SC1 and SC2 of rate \( 1/2 \). A system with 8-PSK constellation and \( \gamma = 0\% \), i.e., the system considered in [16], provides a bandwidth efficiency of 1 b/s/Hz. To maintain this bandwidth efficiency, a system with \( \gamma = 100\% \) (i.e., where all the systematic bits are repeated in Frame 2) needs to use a 16-QAM constellation.

It is clear that with higher-order modulation, the minimum distance must be reduced in order to maintain the same average transmitted power. The reduction of minimum distance in the constellation causes performance loss. However, this loss can be compensated by the diversity gain because more transmitted bits can have a diversity order of 2. As will be shown in this chapter, diversity gain provided by increasing the repetition level overcomes the loss in minimum distance due to the increase of the modulation size \( M \), and results in a power and bandwidth-efficient cooperative scheme over block fading channels.

### 4.2.2 Decoder

With the turbo-like structure of the encoder in Fig. 4.1, two iterative decoding methods can be applied at the receiver. In the first iterative decoding method, described in Fig. 4.2, iterations are only carried out between the two soft-input soft-output (SISO) channel decoders. In the second method, iterations are implemented not only between the two channel decoders but also between the channel decoders and the demappers as illustrated in Fig. 4.3. More specifically, the two demappers simultaneously produce the extrinsic information for the coded bits based on the received signals corresponding to Frame 1 and Frame 2, respectively. Then they are
Figure 4.2 A decoder in the proposed coded cooperative communications: No iterations between the SISO decoders and the $M$-ary demappers.

Figure 4.3 Another decoder in the proposed coded cooperative communications: Iterations are run between the SISO decoders and the $M$-ary demappers.
interleaved before being applied to SISO1 and SISO2. Next, the SISO1 is first activated to produce the extrinsic information for the information bits and the coded bits. The extrinsic information of the coded bits is then interleaved to feedback to demapper 1 for the next iteration [33]. On the other hand, the extrinsic information of the information bits is also interleaved and transferred to SISO2. Finally, similar to SISO1, SISO2 produces both the extrinsic information for the coded bits and information bits to feedback to demapper 2 and SISO1, respectively. In addition, the soft information of the information bits is passed to the decision device to make the hard decisions of the information bits.

The simulation results for the above two decoding methods over the block fading channel are shown in Table 4.1 for the case that both SC1 and SC2 are the same systematic convolutional code of rate 1/2 and generator polynomials (1, 5/7) in octal form. In addition, the target bandwidth efficiency is 1 bit/s/Hz and it is achieved by using 8-PSK signal constellation. Observe that the number of erroneous bits resulted from the second decoding method is smaller than that of the first decoding method when the same number of frames and SNR are set. However, this difference is relatively small in terms of the BER while the complexity of the second decoder is significantly higher than the complexity of the first one.

Another observation based on the simulation results in Table 4.1 is that the BER performance slightly improves from the first iteration to the second iteration in both the decoding methods. For example, a gain of approximately 0.5 dB is achieved for the case that iterations are only carried out between the two channel decoders (Fig. 4.4). However, almost no performance improvement is obtained after the second iteration. Two main reasons are as follows. First, the goal of cooperative communications is to improve the diversity gain in block fading environment. In block fading environment, the fading coefficients are constant over every two frames of the cooperation process. When both the channels from the two users to the destination are in very deep fades, it reduces the instantaneous SNRs of both frames to a very low level, at which iterative decoding is no longer useful. Second, since the interleaving length of a turbo code
Table 4.1  Bit error performance of iterative decoding: RSC codes of rate 1/2 and generator polynomials (1, 5/7), bandwidth efficiency is 1 bit/s/Hz and 8-PSK modulation.

<table>
<thead>
<tr>
<th>Iterations</th>
<th>$E_b^{(1,0)}/N_0 = 5$ dB $F=1500$ frames</th>
<th>$E_b^{(1,0)}/N_0 = 10$ dB $F=3000$ frames</th>
<th>$E_b^{(1,0)}/N_0 = 15$ dB $F=4000$ frames</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17443</td>
<td>7194</td>
<td>1563</td>
</tr>
<tr>
<td>2</td>
<td>16006</td>
<td>5893</td>
<td>1250</td>
</tr>
<tr>
<td>3</td>
<td>15636</td>
<td>5658</td>
<td>1133</td>
</tr>
<tr>
<td>4</td>
<td>15373</td>
<td>5615</td>
<td>1098</td>
</tr>
</tbody>
</table>

(b) Iteration among SISO decoders and demappers.

<table>
<thead>
<tr>
<th>Iterations</th>
<th>$E_b^{(1,0)}/N_0 = 5$ dB $F=1500$ frames</th>
<th>$E_b^{(1,0)}/N_0 = 10$ dB $F=3000$ frames</th>
<th>$E_b^{(1,0)}/N_0 = 15$ dB $F=4000$ frames</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16876</td>
<td>6834</td>
<td>1327</td>
</tr>
<tr>
<td>2</td>
<td>15227</td>
<td>5644</td>
<td>1071</td>
</tr>
<tr>
<td>3</td>
<td>14649</td>
<td>5402</td>
<td>1014</td>
</tr>
<tr>
<td>4</td>
<td>14516</td>
<td>5330</td>
<td>1039</td>
</tr>
</tbody>
</table>
used in cooperative communications needs to be the same as the frame length, which is typically very short, it significantly reduces the error correcting capability of the turbo code.

![Figure 4.4](image)

**Figure 4.4** Error performance with iterations: SC1 and SC2 are RSC codes of rate 1/2 and generator polynomials (1, 5/7), bandwidth efficiency is 1 bit/s/Hz, 8-PSK modulation and $E_b^{(2,0)}/N_0 = E_b^{(1,0)}/N_0$.

With the above investigations and observations, the first decoding method, shown in Fig. 4.2, with only a small number of iterations, is chosen for the proposed coded cooperative system. In particular, all the simulation results shown later in this chapter are obtained after three iterations of decoding.

Let $s_1[n]$, $n = 1, \ldots, L_1$, and $s_2[n]$, $n = 1, \ldots, L_2$, be the $n$th symbols in Frame 1 and Frame 2, where $L_1$ and $L_2$ are the lengths of Frame 1 and Frame 2, respectively. With the assumption of orthogonal transmission of two users signals, the corresponding baseband-equivalent discrete-time received signals are expressed as:

$$r_1[n] = h_{10}[n]s_1[n] + w_{10}[n]$$

(4.4)

$$r_2[n] = h_{20}[n]s_2[n] + w_{20}[n]$$

(4.5)
where \( r_1[n] \) and \( r_2[n] \) are the received signals for User 1 at the destination from User 1 and User 2, respectively. The quantities \( h_{10}[n] \) and \( h_{20}[n] \) are the fading coefficients of the channels from User 1 and User 2 to the destination (also referred to as User 0), which affect the \( n \)th transmitted symbols of the two frames, respectively.

For the assumed block fading environment, the fading coefficients are constant over two frames, i.e., \( h_{10}[n] = h_{10} \) for \( n = 1, \ldots, L_1 \), and \( h_{20}[n] = h_{20}, n = 1, \ldots, L_2 \). Furthermore, both \( h_{10} \) and \( h_{20} \) are modeled as circularly symmetric complex Gaussian random variables (i.e., Rayleigh fading channels) with variances \( E_{s}^{(1,0)}/2 \) and \( E_{s}^{(2,0)}/2 \) per dimension, respectively. User 1’s channel symbols, namely \( \{s_1[n]\} \), are drawn from an \( M \)-ary signal constellation \( \chi^{(1)} \) whose average symbol energy is normalized to unity. Similarly, User 2’s channel symbols, \( \{s_2[n]\} \), belong to an unit-energy \( M \)-ary constellation \( \chi^{(2)} \). The received signals in (4.4) and (4.5) are also disturbed by additive circularly symmetric complex Gaussian noise \( w_{10}[n] \) and \( w_{20}[n] \) of variance \( N_0/2 \) per dimension. Note that with the above notations, the average symbol-energy-to-noise-density ratios for User 1-destination channel and User 2-destination channel are \( E_{s}^{(1,0)}/N_0 \) and \( E_{s}^{(2,0)}/N_0 \), respectively. Similarly, the average symbol-energy-to-noise-density ratios of the inter-user channels, namely User 1-User 2 and User 2-User 1 channels, are denoted as \( E_{s}^{(1,2)}/N_0 \) and \( E_{s}^{(2,1)}/N_0 \), respectively. Furthermore, define the related average information-bit-energy-to-noise-density ratios for the four mentioned channels as \( E_{b}^{(1,0)}/N_0 \), \( E_{b}^{(2,0)}/N_0 \), \( E_{b}^{(1,2)}/N_0 \) and \( E_{b}^{(2,1)}/N_0 \), respectively. The relationship between the average information-bit energy and the corresponding average symbol energy depends on the code rate and the constellation size.

As shown in Fig. 4.2, at the destination, the received signals \( \{r_1[n]\}_{n=1}^{L_1} \) and \( \{r_2[n]\}_{n=1}^{L_2} \) are first applied to the \( M \)-ary demappers. In general, since the decoder at the destination works with both the constellations \( \chi^{(1)} \) and \( \chi^{(2)} \) of the two users, the superscripts of the constellations are therefore dropped in the following discussion. With the assumption that perfect channel state information is available at the destination, each demapper first computes the log-likelihood ratios (LLRs) for all the
coded bits, i.e., the bit metrics, as follows:

\[
L(b_k[n]|r_i[n]) = \log \left( \frac{\sum_{s_i[n] \in \chi_{1,k}} \exp \left( -\frac{|r_i[n] - b_0 s_i[n]|^2}{2N_0} \right)}{\sum_{s_i[n] \in \chi_{0,k}} \exp \left( -\frac{|r_i[n] - b_0 s_i[n]|^2}{2N_0} \right)} \right)
\]

(4.6)

where \( k = 1, \ldots, m = \log_2 M \); \( i = 1, 2 \); \( n = 1, \ldots, L_i \) and \( b_k[n] \) is the \( k \)th coded bit in a group of \( m = \log_2 M \) bits carried by \( s_i[n] \), the \( n \)th symbol of frame \( i \). The subsets \( \chi_{1,k} \) and \( \chi_{0,k} \) contain the signal points in the \( M \)-ary constellation whose \( k \)th labelling bits are “1” and “0”, respectively.

After bit-metric computation is fulfilled, the bit-metric sequence is deinterleaved and demultiplexed to separate the systematic-bit metrics from the parity-bit metrics. Next, the systematic-bit metric sequences produced by the two demappers are combined before applied to the SISO decoders. The SISO1 produces the extrinsic information for the systematic bits (indicated by the symbol \( L_e \) in Fig. 4.2), which is then passed to the SISO2 as shown in Fig. 4.2. The extrinsic information provided by SISO1 is used in SISO2 as the \textit{a priori} information for the systematic-bit stream (indicated by \( L_a \) in Fig. 4.2). The soft output of SISO2 is used to make the final hard decision on the original systematic bits. In addition, the SISO2 also produces the extrinsic information on information bit to feedback to the SISO1 after it is de-interleaved. This information is used as a \textit{a priori} information for the systematic-bit stream at the input of the SISO1 in the next iteration. The functionality of the SISO module is thoroughly discussed in [34]. As discussed above, almost no performance improvement is obtained after the third iteration. Therefore, only 3 iterations are used at the receiver side in our investigation.

### 4.3 Performance Analysis

Since the proposed coded cooperative communications system is based on bit-interleaved coded modulation (BICM), to determine its error performance, one needs to first obtain the pairwise error probability (PEP) of \( M \)-ary BICM over a point-to-point block fading channel. Although the PEP of \( M \)-ary BICM is determined here for the maximum likelihood decoding [35], the analytical results shall be compared
with the simulation results of the iterative decoding system in Fig. 4.2. This is reasonable as it is well-known that the performance of the suboptimal iterative decoding approaches that of the maximum likelihood decoding [28].

4.3.1 Pairwise Error Probability for Point-to-Point M-ary BICM

Let \( r = (r[1], r[2], \ldots, r[L]) \) be the received vector corresponding to an arbitrary transmitted frame and \( h \) be the fading coefficient of that frame. If \( s = (s[1], s[2], \ldots, s[L]) \) is the transmitted frame of the coded symbols, then

\[
\mathbf{r} = h \mathbf{s} + \mathbf{w}
\]

(4.7)

where \( \mathbf{w} = (w[1], w[2], \ldots, w[L]) \) is the vector of i.i.d. circularly symmetric Gaussian random variables.

The pairwise error probability (PEP) for a coded system is defined as the probability of deciding in favor of a codeword \( \hat{\mathbf{b}} = (\hat{b}[1], \hat{b}[2], \ldots, \hat{b}[N]) \) when the codeword \( \mathbf{b} = (b[1], b[2], \ldots, b[N]) \) was actually transmitted. For M-ary BICM with an ideal interleaver it can be shown that [32] the PEP only depends on the Hamming distance \( d \) between \( \mathbf{b} \) and \( \hat{\mathbf{b}} \), the constellation \( \chi \) and the mapping scheme \( \mu \). It can be expressed as [35]

\[
\text{PEP}(d, \chi, \mu) = P\left[ P(\hat{b}|\mathbf{r}) > P(b|\mathbf{r}) \mid \mathbf{b} \right]
\]

\[
= P\left[ \sum_{j: \hat{b}[j] \neq b[j]} \log \left( \frac{P(\hat{b}[j]|r[n_j])}{P(b[j]|r[n_j])} \right) > 0 \mid \mathbf{b} \right] \quad (4.8)
\]

where \( r[n_j] \) is the \( n_j \)th received signal that carries the \( j \)th bit in the transmitted codeword.

It follows from that (4.7) the received signal vector \( \mathbf{r} \) depends on the noise and fading realizations \( \mathbf{w} \) and \( h \), respectively. Furthermore, it also depends on the particular modulation symbols \( \mathbf{s} \), the bit position \( k \) of the binary label. For convenience, these random variables are grouped in a random vector \( \mathbf{\nu} \triangleq (\mathbf{w}, h, \mathbf{s}, k) \). Obviously
\( \nu \) depends on the signal constellation \( \chi \) and the mapping scheme \( \mu \). Without loss of generality, assume that \( b \) and \( \hat{b} \) differ in the first \( d \) positions. Then the PEP can be rewritten as

\[
\text{PEP}(d, \chi, \mu) = P \left( \sum_{j=1}^{d} \Lambda_j > 0 \right) \tag{4.9}
\]

where the new random variable \( \Lambda_j \), called the \textit{a posteriori} log-likelihood ratio, is defined as

\[
\Lambda_j = \log \frac{P(\hat{b}[j] = \overline{b}|\nu)}{P(\hat{b}[j] = b|\nu)} \tag{4.10}
\]

where \( b \) is either 0 or 1 and \( \overline{b} = 1 - b \). The \textit{a posteriori} probability in (4.10) is computed as follows:

\[
P[\hat{b}[j] = b|\nu] \propto \sum_{s \in \chi_{b,k}} \exp(-|r[n_j] - hs[n_j]|^2) \tag{4.11}
\]

where \( \chi_{b,k} \) is the subset of all signal points with the \( k \)th binary position equal to \( b \).

The computation of (4.9) is obtained in [35] using the saddle point method. In particular, it is shown in [35] that determining the tail probability in (4.9) can be conveniently carried out by using the cumulant transform of the random variable \( \Lambda_j \) instead of its probability density function. The cumulant transform of \( \Lambda_j \) is defined as

\[
\kappa_j(\omega) \triangleq \log \mathbb{E}[e^{\omega \Lambda_j}] = \log \mathbb{E}_\nu \left[ \left( \frac{P(\hat{b}[j] = \overline{b}|\nu)}{P(\hat{b}[j] = b|\nu)} \right)^\omega \right] \tag{4.12}
\]

where \( \mathbb{E}_\nu \) denotes the expectation over all the random variables in \( \nu \). This expectation can be efficiently computed by using the Gauss-Hermite approximation method in [36].

The PEP calculation using the cumulant transforms is [35]

\[
\text{PEP}(d, \chi, \mu) = \frac{1}{\sqrt{2\pi \sum_{j=1}^{d} \kappa_j''(\hat{\omega})}} \exp \left( \sum_{j=1}^{d} \kappa_j(\hat{\omega}) \right) \tag{4.13}
\]

where \( \hat{\omega} \) is the solution of \( \kappa_j'(\hat{\omega}) = 0 \) and it is called the saddle value. It can be proved that this value exists and is unique. For the binary-input output-symmetric channel,
the saddle point is placed at \( \hat{\omega} = \frac{1}{2} \). Therefore \( \kappa_j(\hat{\omega}) \) and \( \kappa_j''(\hat{\omega}) \) in (4.13) are given as [35]

\[
\kappa_j(\hat{\omega}) = \log \mathbb{E}_\nu \left[ \frac{P(b[j] = \overline{b}| \nu)}{P(b[j] = b| \nu)} \right] \\
\kappa_j''(\hat{\omega}) = \frac{1}{\mathbb{E}[e^{\kappa_j(\hat{\omega})}]} \mathbb{E} \left[ \left( \frac{\log P(b[j] = \overline{b}| \nu)}{P(b[j] = b| \nu)} \right)^2 \sqrt{\frac{P(b[j] = \overline{b}| \nu)}{P(b[j] = b| \nu)}} \right] 
\]

(4.14)

(4.15)

Recall that the PEP in (4.13) is for the point-to-point BICM communications system. It is applied in the next subsection to derive the end-to-end BEP for the proposed BICM coded cooperative systems.

### 4.3.2 End-to-End Bit Error Probability

As thoroughly discussed in [37], the end-to-end bit error probability for the coded cooperative systems is also a function of the inter-user channel quality. In other words, it depends on whether or not decoding of the first frame is successful at each user. Similar to [37], define a new random variable \( \Theta \) that represents the events at both users in decoding the first frames of their partners. In particular, the outcomes of this random variable are defined as follows.

- \( \Theta = 1 \) when both User 1 and User 2 successfully decode the first frames of their partners.
- \( \Theta = 2 \) when User 1 fails to decode the first frame of User 2 and User 2 succeeds in decoding the first frame of User 1.
- \( \Theta = 3 \) when User 1 succeeds in decoding the first frame of User 2 but User 2 fails to decode the first frame of User 1.
- \( \Theta = 4 \) when both User 1 and User 2 fail to decode the first frames of their partners, respectively.
Conditioned on $\Theta$, the bit error probability (BEP) can be expressed as

$$P(\text{bit error}|\Theta) = \int \int \min \left[ 1, \frac{1}{k_c} \sum_{d=d_{\text{free}}}^{\infty} C_d \text{PEP}^\text{coop}(d, \chi, \mu, \gamma, \xi_1, \xi_2|\Theta) \right] f(\xi_1) f(\xi_2) d\xi_1 d\xi_2$$

(4.16)

In (4.16) $d_{\text{free}}$ is the free distance of the convolutional code, $\xi_1 = |h_1|^2/N_0$ and $\xi_2 = |h_2|^2/N_0$ are the instantaneous signal-to-noise ratios of User 1-destination channel and User 2-destination channel, respectively. The parameter $k_c$ is the number of information bits per branch in the code trellis and $C_d$ is the number of bits in error corresponding to the error event on the code trellis that has Hamming weight $d$. The functions $f(\xi_1)$ and $f(\xi_2)$ are the probability density functions of the random variables $\xi_1$ and $\xi_2$, respectively. Since Rayleigh fading channels are considered, $f(\xi_1)$ and $f(\xi_2)$ are exponential distributions as expressed in Equation (2.9). The function min($\cdot$) in (4.16) is used to obtain the tight bound [38, 39] on the BEP. Note that the pairwise error probability in a cooperative system, $\text{PEP}^\text{coop}$ in (4.16), is explicitly shown to depend not only on $d, \chi, \mu$; but also on $\rho, \gamma, \xi_1$ and $\xi_2$.

The $\text{PEP}^\text{coop}$ expressions for User 1 corresponding to the above four specific outcomes of $\Theta$ are provided in the following. Similar expressions hold for User 2.

**Case 1 ($\Theta = 1$):** Recall that in this case both users are successful in decoding the first frames of their partners. This means that one codeword is delivered to the destination by two sub-codewords on two independent paths, thus a diversity order of 2 is obtained. Assume that the Hamming weight of each sub-codeword is proportional to the frame length, then the conditional PEP is:

$$\text{PEP}^\text{coop}(d, \chi, \mu, \rho, \gamma, \xi_1, \xi_2|\Theta = 1) = \frac{\exp(d_1 k(\hat{\omega}|\xi_1) + d_2 k(\hat{\omega}|\xi_2))}{\sqrt{2\pi(d_1 k''(\hat{\omega}|\xi_1) + d_2 k''(\hat{\omega}|\xi_2))}\hat{\omega}}$$

(4.17)

where $d_1$ and $d_2$ are the Hamming weights of Frame 1 and Frame 2, respectively. In the cooperative system without using repetition code [11–14] the summation of $d_1$ and $d_2$ equals to the Hamming weight $d$ of the full codeword. Whereas in our proposed system, $d_1$ and $d_2$ depend not only on the cooperative level $\rho$ but also on
the repetition level $\gamma$ and the code rate $r_c$. In particular,

$$d_1 \approx (1 - \rho)[2 - (1 - \gamma)r_c]d$$  \hspace{1cm} (4.18)

$$d_2 \approx \rho[2 - (1 - \gamma)r_c]d$$  \hspace{1cm} (4.19)

**Case 2 ($\Theta = 2$):** In this case, the second frame of User 1 is produced by User 2 on the basis of the decoded results of the first frame of User 1. Therefore,

$$\text{PEP}^{\text{coop}}(d, \chi, \mu, \rho, \gamma, \xi_1, \xi_2 | \Theta = 2) = \frac{\exp(d_1\kappa(\hat{\omega} | \xi_1) + d_2\kappa(\hat{\omega} | \xi_1) + d_2\kappa(\hat{\omega} | \xi_2))}{\sqrt{2\pi(d_1\kappa''(\hat{\omega} | \xi_1) + d_2\kappa''(\hat{\omega} | \xi_1) + d_2\kappa''(\hat{\omega} | \xi_2))}} \hat{\omega}$$  \hspace{1cm} (4.20)

**Case 3 ($\Theta = 3$):** In this case, the destination decodes just the first frame received directly from User 1. Hence,

$$\text{PEP}^{\text{coop}}(d, \chi, \mu, \rho, \gamma, \xi_1, \xi_2 | \Theta = 3) = \frac{\exp(d_1\kappa(\hat{\omega} | \xi_1))}{\sqrt{2\pi(d_1\kappa''(\hat{\omega} | \xi_1))}}$$  \hspace{1cm} (4.21)

**Case 4 ($\Theta = 4$):** This case implies no cooperation between the two users and

$$\text{PEP}^{\text{coop}}(d, \chi, \mu, \rho, \gamma, \xi_1, \xi_2 | \Theta = 4) = \frac{\exp(d\kappa(\hat{\omega} | \xi_1))}{\sqrt{2\pi(d\kappa''(\hat{\omega} | \xi_1))}}$$  \hspace{1cm} (4.22)

Finally, the end-to-end bit error probability is obtained by averaging over all four cases:

$$P_b = \sum_{\Theta=1}^{4} P(\text{bit error} | \Theta)P(\Theta)$$  \hspace{1cm} (4.23)

where the probability distribution $P(\Theta)$ of the random variable $\Theta$ can be determined as in [37]. For completeness, Appendix A summarizes the derivation of $P(\Theta)$ from the inter-user channel SNR.

### 4.4 Results and Discussions

We first consider a cooperative system using two identical SC1 and SC2 with rate-1/2, 4-state RSC codes of the same generator polynomials (1; 5/7) in octal form, and information block length of 132 bits. For the target bandwidth efficiency of 1 b/s/Hz, we select 2 extreme repetition levels: (i) $\gamma = 0\%$ with 8-PSK, which
Figure 4.5  Performance of the proposed coded cooperative systems with bandwidth efficiency of $1/b/s/\text{Hz}$, $E_b^{(2,0)}/N_0 = E_b^{(1,0)}/N_0$. 

is equivalent to the system in [16] and (ii) $\gamma = 100\%$ with 16-QAM. Perfect inter-user signaling is assumed. Fig. 4.5 presents the BER performance versus the same average signal-to-noise ratio, $E_b/N_0$, obtained by simulations and numerical analysis, over independently and identically Rayleigh-distributed user channels, where $E_b$ is the energy per information bit. Similar to [37], we truncate the summations in the computations of the union bounds in (A.2) and (4.16) to the first 15 terms of $A_d$ and $C_d$, respectively. As can be observed, the simulation results are quite close to the analytical results derived in Section 4.3. Our investigation shows that if all the terms of $A_d$ and $C_d$ are included in (A.2) and (4.16) then the upper bounds are within 2 to 4 dB compared to the simulation results. This is also similar to what observed in [37] and [39] for the case of binary modulation.

Also shown in this figure is the error performance of the non-cooperative system that uses the same RSC code and 8-PSK, and hence has the same spectral efficiency. It is not surprising to see that significant coding gains are achieved by the proposed
Figure 4.6 Performance of the proposed coded cooperative systems with bandwidth efficiency of $1/b/s/Hz$, and $E_b^{(2,0)}/N_0 = E_b^{(1,0)}/N_0 + 5$ dB.

coded cooperative systems over the non-cooperative one. But what is interesting to observe is that a large coding gain (of about 2 dB at BER of $10^{-4}$) is realized by increasing $\gamma$ from 0% to 100% in the proposed system. This observation clearly illustrates the capability of achieving a higher diversity gain in the proposed system by repeating more systematic bits.

The results shown in Fig. 4.6 are obtained for the same systems as in Fig. 4.5, but with unequal average signal-to-noise ratios: the SNR of User 2 at the destination is 5 dB better than that of User 1. This can happen if User 2 is at the location closer to the destination than User 1. Other possibility is when the transmission link from User 2 to the destination has fewer obstructions than the link from User 1 to the destination. Observe that the coding gain between the proposed systems with different repetition levels is still maintained in this case. However, compared to the non-cooperative system, the error performance of the proposed systems improves
quite significantly. For example, at the BER of $10^{-3}$, the coding gain of the proposed system with $\gamma = 100\%$ over the non-cooperative system increases from 10 dB in Fig. 4.5 to 13 dB in Fig. 4.6. This observation means that User 1 obtains more benefit from the cooperation with User 2, who has a better channel to the destination.

The end-to-end BER performance depends not only on the quality of the channel from each user to the destination but also on the inter-user channel quality. As mentioned in Section 4.3, the effect of inter-user channel quality on the end-to-end BER performance is quantified by the probability distribution of the random variable $\Theta$. As an illustrative example, consider the case to obtain a bandwidth efficiency of 1 b/s/Hz as in Fig. 4.5, but with various inter-user channel quality levels indicated by its average information-bit-energy-to-noise-density ratio, $E_{b}^{(1,2)}/N_0$.

Table 4.2 tabulates $P(\Theta)$ for different values of $E_{b}^{(1,2)}/N_0$ and $\{\gamma, \rho\}$ and Fig. 4.7 shows the corresponding bit error rates of the proposed system. Note that for a given
Table 4.2 Probability distributions of $\Theta$, $P(\Theta)$, for different values of $E_b^{(1,2)}/N_0$.

<table>
<thead>
<tr>
<th>$E_b^{(1,2)}/N_0$ (dB)</th>
<th>$\Theta = 1$</th>
<th>$\Theta = 2$</th>
<th>$\Theta = 3$</th>
<th>$\Theta = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.517</td>
<td>0.202</td>
<td>0.202</td>
<td>0.079</td>
</tr>
<tr>
<td>20</td>
<td>0.935</td>
<td>0.03</td>
<td>0.03</td>
<td>0.005</td>
</tr>
<tr>
<td>25</td>
<td>0.980</td>
<td>0.01</td>
<td>0.01</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>30</td>
<td>0.996</td>
<td>0.002</td>
<td>0.002</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>40</td>
<td>0.999</td>
<td>0.0005</td>
<td>0.0005</td>
<td>$\approx 0$</td>
</tr>
</tbody>
</table>

(a) $\gamma = 0\%$, $\rho = 33\%$

<table>
<thead>
<tr>
<th>$E_b^{(1,2)}/N_0$ (dB)</th>
<th>$\Theta = 1$</th>
<th>$\Theta = 2$</th>
<th>$\Theta = 3$</th>
<th>$\Theta = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.376</td>
<td>0.237</td>
<td>0.237</td>
<td>0.149</td>
</tr>
<tr>
<td>20</td>
<td>0.896</td>
<td>0.05</td>
<td>0.05</td>
<td>0.004</td>
</tr>
<tr>
<td>25</td>
<td>0.966</td>
<td>0.017</td>
<td>0.017</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>30</td>
<td>0.994</td>
<td>0.003</td>
<td>0.003</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>40</td>
<td>0.999</td>
<td>0.0005</td>
<td>0.0005</td>
<td>$\approx 0$</td>
</tr>
</tbody>
</table>

(b) $\gamma = 100\%$, $\rho = 50\%$. 
pair of curves that have the same marker, the solid curve is obtained from the analysis while the dashed curve represents the simulation results. At first, the simulation and analytical results are in a good agreement. In general, the proposed scheme provides a better performance than the non-cooperative system for a wide range of signal-to-noise ratios over both the user-destination and inter-user channels. This performance gain increases with the increased inter-user channel quality (shown in Fig. 4.7), corresponding to the increased $P(\Theta = 1)$ as shown in Table 4.2. When the inter-user channel’s $E_b^{(1,2)}/N_0$ is about 40 dB, the end-to-end performance approaches the performance corresponding to the ideal inter-user channel. Fig. 4.7 also indicates that for a very poor inter-user channel quality, e.g., $E_b^{(1,2)}/N_0 = 10$ dB, the BER performance of the proposed system is better with the lower repetition level. This can be explained by the higher fraction in the cooperative mode, e.g., 70% for $\gamma = 0\%$ as compared to 60% for $\gamma = 100\%$. As the inter-user channel quality improves, the BER performance of the proposed system with a higher repetition level becomes better. For example, the system BER performance with $\gamma = 100\%$ approaches that with $\gamma = 0\%$ for $E_b^{(1,2)}/N_0 = 20$ dB, and is better for $E_b^{(1,2)}/N_0 = 25$ dB. As $E_b^{(1,2)}/N_0$ is further increased, the system BER performance with $\gamma = 100\%$ is much better than that with $\gamma = 0\%$. The results in Fig. 4.7 are useful for selecting the optimum value of $\gamma$ based on the available average inter-user channel quality.

Next, consider the bandwidth efficiency of 2 b/s/Hz. Here SC1 and SC2 are selected to be 4-state, rate-2/3 RSC codes of the same parity check sequences $h^{(0)} = 7$, $h^{(1)} = 3$, $h^{(2)} = 5$ in octal form. The information block length of $K = 160$ bits is used. To achieve this spectral efficiency, the non-cooperative system and the proposed system with $\gamma = 0\%$ can use a 16-QAM constellation. On the other hand, the proposed systems with $\gamma = 50\%$ and $\gamma = 100\%$ need to employ 32-QAM and 64-QAM constellations, respectively.

The BER performance results presented in Fig. 4.8 shows that the proposed system with $\gamma = 0\%$ only slightly outperforms the non-cooperative system. This observation is quite different from what observed from Fig. 4.5 where the gap between
Figure 4.8 Performance of the proposed coded cooperative systems with bandwidth efficiency of 2 b/s/Hz, $E_b^{(1,0)}/N_0 = E_b^{(2,0)}/N_0$.

these two systems is much larger (approximately 8 dB). The explanation is as follows. In order to obtain the higher bandwidth efficiency of 2 b/s/Hz, a higher code rate of $2/3$ needs to be used. When only the additional parity bits are sent in Frame 2 in the proposed system ($\gamma = 0\%$), the cooperative level is only 25%, whereas it is 33% in the case of Fig. 4.5. This low cooperative level is thus not good enough to boost the overall system performance in the block fading environment. In fact it can be seen that the diversity orders of the two systems are very similar (indicated by the similar slopes of the BER curves). This fact also confirms that repeating more information symbols (i.e., the systematic bits) in block fading environment is critical.

The situation is, however, completely different for the proposed systems that implement higher levels of repetition, namely $\gamma = 50\%$ and $\gamma = 100\%$. As shown in Fig. 4.8, the BER performance of both systems significantly improves over that of the previous two systems. In particular, at the BER level of $10^{-3}$, a coding gain of about 8 dB can be obtained by the proposed system with $\gamma = 50\%$ over the non-cooperative
An additional gain of about 1 dB is possible by increasing $\gamma$ from 50% to 100%. It is also interesting to observe from the slopes of the BER curves in Fig. 4.8 that the diversity orders of the proposed systems with $\gamma = 50\%$ and $\gamma = 100\%$ are also very similar. This can be expected from the fact that the cooperative levels of $\rho = 40\%$ and $\rho = 50\%$ in two systems are quite close.

Figure 4.9 Performance of the proposed coded cooperative systems with bandwidth efficiency of $2\,b/s/Hz$, $E_b^{(2,0)}/N_0 = E_b^{(1,0)}/N_0 + 5\,dB$.

Similar observations are obtained from Fig. 4.9, where the same performance comparisons are examined, but for case of asymmetric channels. The performance improvement of User 1 again illustrates that there is better benefit to cooperate with the user that has a better channel to the destination.

Performance comparisons of the proposed coded cooperative systems with various bandwidth efficiencies and different repetition levels are provided in Fig. 4.10. The first important observation is that it is always advantageous to repeat more information bits in the second frame at any considered bandwidth efficiency. The second observation is that the system with a lower spectral efficiency generally performs bet-
Figure 4.10 Performance comparison with different bandwidth efficiencies, $E_b^{(2,0)}/N_0 = E_b^{(1,0)}/N_0$.

It is also of interest to point out that the system corresponding to bandwidth efficiency of $0.33$ b/s/Hz and $\gamma = 0\%$ uses BPSK and it is exactly the coded cooperative system considered in [16]. As shown in Fig. 4.10, this bandwidth efficiency can be achieved with the proposed system that uses BPSK in Frame 1 and QPSK in Frame 2 to implement the repetition level of $\gamma = 100\%$. The coding gain obtained by deploying such a system over the one in [16] is about $1.5$ dB at the BER level of $10^{-4}$.

Adaptive modulation and coding (AMC) schemes [10] have been proposed for efficient uses of the communications systems’ resources (e.g., power and bandwidth) in a time-varying environment. In addition to the conventional AMC parameters of coding rate and constellation size, the proposed coded cooperative scheme also
introduces the new and effective parameter, repetition level, that gives finer granularity, robust and smooth control. Changing the repetition level is relatively simple without switching encoders and signal mappers at neither the transmitter nor the receiver. Furthermore, changing coding rate and constellation size introduces coarse steps in both SNR and bandwidth efficiency. Hence, repetition level is more effective in providing smooth and robust control for fine-granularity adaptation to small environmental variations. As an illustrative example, Fig. 4.11 shows the bit error performance versus SNR curves for various repetition levels but with the fixed coding rate and constellation size. It is observed that the higher the repetition level is, the better the error performance is obtained. To achieve an example target BER of $10^{-3}$, changing the repetition level from 0 to 1 can adapt to an $E_b/N_0$ range from 15 dB to 17 dB (in Fig. 4.11) for a provision of bandwidth efficiency range from 1 to 1.32 b/s/Hz as shown in Fig. 4.12.
Figure 4.12 Adaptive bandwidth efficiency versus $E_b^{(2,0)} / N_0 = E_b^{(1,0)} / N_0$ with fine granularity.

4.5 Summary

A novel bandwidth-efficient coded cooperative system was proposed. Using a very accurate and low complexity bounding technique originally developed for bit-interleaved coded modulation, the performance analysis of the proposed system is also provided to verify the performance advantages of the proposed system. In particular, it was demonstrated that using higher-order modulation not only can provide a higher bandwidth efficiency but also a means of increasing the repetition level of the systematic bits. The latter was found to be critical to improve the overall system performance in block fading environment. In turn, the repetition level $\gamma$ can be used as the control parameter when an adaptive system is considered to increase the system throughput.

However, the adverse effect of block fading is not completely removed with the cooperative communications. That is, burst errors still happen when both the uplink channels are in deep fade. The effect of burst errors can be reduced by using multiple partner cooperative protocols. But, this option makes wireless terminals and
the partner choice problem, which is handled by the medium access control at the
destination, more complicated. In the next chapter, another solution to deal with the
burst error phenomenon will be proposed. The proposed scheme not only combats
with the burst error but also increases the transmission rate of the wireless commu-
nications which is the main issue of data applications in the next generation wireless
networks [6].
5. Cooperative Coding with Hybrid-ARQ Soft Combining in Block Fading Environment

5.1 Introduction

As demonstrated in the previous chapter, although cooperative transmission in coded systems can significantly improve the system performance in block fading environments compared to the non-cooperative counterparts, when both the uplink channels in pairwise cooperative transmission are in deep fade, burst errors occur. In this situation, it is difficult for the receiver to restore the users’ information. This severely degrades the quality of service. The automatic retransmission request (ARQ) protocol in which time diversity is efficiently utilized can also be a good candidate for solving this problem.

In the basic ARQ protocol, the receiver detects frame errors and requests retransmission of the unsuccessfully received packets. This leads to a very powerful time diversity scheme and allows the system to operate at relatively high frame error rate (FER) because the missing packets can be delivered as part of retransmission protocol [5]. In order to provide a higher throughput and a lower latency in packet transmission, hybrid-ARQ (HARQ) schemes are designed to combine the ARQ protocols with forward error correcting (FEC) codes such as convolutional codes and turbo codes. In this approach, the average number of retransmission is reduced due to the error correcting capability of the FEC codes. The HARQ scheme can be implemented in different ways depending on how the means by which we form transmission and retransmission packets and decode the original information based on retransmitted packets at the destination. This chapter focuses only on the HARQ with soft com-
binning scheme, where each retransmission packet carries different parity bits and the currently received packet is combined with the previous ones to form lower rate and stronger codes at the receiver side.

The HARQ protocols have been considered in [40–45] for point-to-point communications. In [46–48], the HARQ transmission protocols for the relay-based network have been proposed, where the broadcast-oriented nature of radio transmission is taken into account to improve the system performance. Relay-based networks provide a promising solution for current and future wireless networks due to some dominant benefits such as low cost of deployment is low and the fact that there is no need to re-plan the radio frequencies because the relays can be designed to operate at non-licensed frequencies, i.e., 5 GHz [49]. In particular, a source broadcasts its first information block to all the relays. Then the transmission in the next frame is carried out by either all the relays which successfully decode the source’s information, or only by one relay which has correct decoded information and the most favorable fading condition to the destination.

In our proposed scheme, only one partner, which is already chosen by the destination [31], receives, decodes and relays the information to the destination to save the power assumption on signal processing for other terminal users and to keep the complexity of the terminal users at a reasonable level. In addition to the bit error performance improvement, the HARQ transmission offers a high system throughput. This is because when the channel gain is good, the destination will successfully decode the information based on just the first or the second transmission attempt. Therefore, the transmission process of a given information packet is likely to terminate before a given threshold (such as the maximum number of retransmission packets or the round trip delay) is reached. This means that a high code rate is obtained. On the other hand, when the channel condition is so severe that it causes the received information errors, the destination will require the transmitters to transmit additional coded packets to increase the incremental redundancy. As a result, the destination can correctly decode the received information and the code rate used in this case is
low. The overall code rate is still higher than using the fixed coding rate to obtain the same bit error rate level [5].

Recently, the HARQ protocol has also been proposed for cooperative communications using rate-compatible punctured convolutional (RCPC) codes [50]. In particular, the HARQ protocol in [50] has been designed for the situation that the fading condition unchanges over the whole HARQ transmission process of one information frame. It means that no time diversity can be utilized by this protocol. Therefore, only a spatial diversity order of 2 is obtained for the pairwise cooperative system. Compared to the cooperative system that does not implement HARQ transmission, this protocol improves the system throughput and reduces the effect of poor inter-user channel on the system performance. However, the protocol requires variable transmission frame length, which imposes a high demand on the signalling procedure to assist the transmission and reception. Furthermore, due to the transceiving and processing delay of the protocol, it might be difficult to implement it in the real world.

In contrast to [50], our proposed HARQ soft combining protocol shall be designed for the coded cooperative system under the condition that the length of each coded frame is equivalent to the coherence time, $T_c$, of the wireless channel. This implies that each fading coefficient is constant over only one frame duration rather than the whole HARQ round trip [50] and it changes independently from one frame to the next. Perfect channel state information available at the receiver for coherence detection is also assumed in this work. Furthermore, the proposed HARQ protocol is designed for the coded systems in which convolutional codes are used instead of RCPC codes. Convolutional codes are preferred to RCPC codes here due to the simplicity of implementation and the constant transmission frame length.

The remainder of this chapter is organized as follows. The proposed transmission scheme based on coded cooperative communications and the HARQ soft combining protocol is introduced in Section 5.2. The throughput of this proposed transmission scheme is analyzed in Section 5.3. The end-to-end bit error performance of the scheme is also investigated in this section. Section 5.4 presents numerical results and
Figure 5.1 Three-node cooperation topology (Similar to Fig. 3.1).

discussions. Section 5.5 concludes the chapter.

5.2 Hybrid-ARQ Protocol in Pairwise Coded Cooperative Communications

Consider the HARQ scheme for the pairwise cooperative coded system as plotted in Fig. 5.1. The original information bit stream of each user is first augmented by cyclic redundancy check (CRC) bits for error detection purpose, and then applied to a convolutional encoder as shown in Fig. 5.2. For an input sequence \( I(x) \) of \( K \) information and CRC bits, the output sequences \( C_1(x) \) of \( (K + m) \) bits and \( C_2(x) \) of \( (K + m) \) bits, are alternatively chosen to transmit in the HARQ protocol where \( m << K \) accounts for a small number of termination bits. Assume that \( C_1(x) \) is chosen to transmit at the initial step. If the destination receiver fails to decode information based on the received signal corresponding to \( C_1(x) \) and requests retransmission, the transmitter will send \( C_2(x) \) to the receiver. After receiving the second sequence, the receiver will combine the first and second received sequences for decoding. After this stage, if the transmitter still receives request for retransmission, it will resend \( C_1(x) \), then \( C_2(x) \) and so on. Consider the HARQ scheme applied in a simple cooperative
network with 3 users where User 1 (source) sends data to User 0 (destination) and User 2 is the partner of User 1. At first, User 1 sends the \((K + m)\)-bit frame, \(C_1(x)\), to the destination and its partner. Both destination and partner decode the received frame from the source and detect its CRC bits. If User 1’s information is successfully decoded by the destination (i.e., no error is detected) then the destination will send an ACK (positive acknowledge) message to both User 1 and User 2, indicating the successful reception.

In the case that errors are detected, the destination will send an NACK (negative acknowledge) message to both User 1 and User 2, requesting for retransmission. In this case, there are the following two possibilities depending on the error detection of User 1’s frame at User 2’s (partner) receiver:

- **Non-cooperative retransmission mode**: If User 2 (partner) fails in CRC detection, it will send an NACK message to both User 1 (source) and User 0 (destination), indicating that it cannot participate in cooperative retransmission. In this situation, User 1 (source) will send the complete \((K + m)\)-bit frame \(C_2(x)\) to both User 0 (destination) and User 2 (partner).

- **Cooperative retransmission mode**: If User 2 (partner) successfully detects the received \((K + m)\)-bit frame \(C_1(x)\) from User 1, it will send an ACK message to both User 1 (source) and User 0 (destination), indicating that it will participate in cooperative retransmission. In this situation, User 1 (source) will send one half of the \((K + m)\)-bit frame \(C_2(x)\) while and User 2 (partner) will collaborate by sending the other half of the \((K + m)\)-bit frame \(C_2(x)\) of User 1.

It can be seen that, for both possibilities described above, User 0 (destination)
receives only one \( (K + m) \)-bit frame, \( C_2(x) \), of User 1, either from only User 1 (in non-cooperative retransmission mode) or from both Users 1 and 2 (in cooperative retransmission mode). Subsequently, User 0 (destination) combines both the received \( (K + m) \)-bit frame \( C_1(x) \) and \( (K + m) \)-bit frame \( C_2(x) \) for further soft decoding and detection. In other words, the proposed HARQ scheme for the pairwise coded cooperative system uses an equivalent amount of retransmitted bits as in a traditional point-to-point HARQ scheme. However, the cooperative retransmission mode introduces extra spatial diversity, which can improve the link quality.

Next, if User 0 (destination) successfully decodes both the received \( (K + m) \)-bit frame \( C_1(x) \) and \( (K + m) \)-bit frame \( C_2(x) \), it will send an ACK message to both User 1 and User 2, declaring the successful reception. Otherwise, if it fails in decoding the received frames, it will request for further retransmission. Depending on its decoding status, User 2 (partner) will decide on its participation in cooperative retransmission as follows.

- For an HARQ round trip of a given information packet, if User 2 (partner) has been involved in the cooperative retransmission mode, it will continue to do so until it receives the positive ACK from the destination or the number of retransmission packets reaches the threshold, regardless of its CRC detection status.

- If the previous retransmission is in a non-cooperative mode (i.e., only the source was involved), User 2 (partner) will check the status of its CRC detection to select one of the 2 following possibilities:
  - If its CRC detection is successful, User 2 (partner) will send an ACK message to both User 1 (source) and User 0 (destination), indicating that it will participate in the cooperative retransmission mode. In this situation, User 1 (source) will send one half of the retransmission frame while User 2 (partner) will collaborate by sending the other half.
  - If its CRC detection fails, User 2 (partner) will send an NACK message to
both User 1 (source) and User 0 (destination), indicating that it will not participate in the retransmission. In this non-cooperative retransmission mode, only User 1 (source) will send the complete retransmission frame.

The HARQ transmission process stops whenever no error is detected at the destination (by means of CRC) or the number of transmission frames for a given information frame reaches a preset threshold value $T$. It is noted that only when User 1 and User 2 receive the ACK message related to a given information frame, coded frames related to that information frame in users’ buffers will be discarded. The preset threshold on the number of transmission frames is selected on the basis of tolerable delay in retransmission and limited buffer size at the wireless terminals. It is also worth pointing out that the feedback ACK/NACK message can be indicated by only one bit and sent over a dedicated signal channel or embedded on the next transmission frame in a well-protected error-free manner.

5.3 Performance Analysis of the Hybrid-ARQ Protocol in Coded Cooperative Communications

5.3.1 Tree Representation of the Hybrid-ARQ Protocol

Although the analytical framework can be generalized to any arbitrary value of $T$, for simplicity of presentation and illustration, set the threshold value on the number of transmission frames $T = 4$ in the following discussions.

Let $OK_{12}^{(j)}$ be the event that User 2 successfully decodes its partner information, where the subscript 12 implies that this event depends on the quality of the channel condition between User 1 and User 2. The superscript $j$ means that User 2 succeeds in decoding information after the $j$th transmission attempt of User 1 (Source). Conversely, $OK_{12}^{(j)}$ represents the event that User 2 fails to decode its partner information.

Fig. 5.3 shows the tree representation used for conducting the throughput and bit error performance analysis of the proposed HARQ protocol with $T = 4$. Each level in the tree represents all the possibilities of one transmission attempt in the HARQ
Figure 5.3 Tree representation of the proposed cooperative HARQ process with $T = 4$.

protocol. Note that the tree in Fig. 5.3 has four levels corresponding to $T = 4$. Each branch of the tree indicates one possible transmission (or retransmission) for a given information frame at each level, and is labeled by the names of the involved users.

In particular, a dashed branch represents a transmission (or retransmission) in a non-cooperative mode by only User 1 (source) while a solid branch implies a retransmission in a cooperative mode involving both User 1 (source) and User 2 (partner). As previously discussed, the selected retransmission mode at each level may depend on the error detection status of User 2 (partner). In such a case, the corresponding probability $P(OK_{12}^{(j)})$ or $P(\overline{OK}_{12}^{(j)})$ is also attached to each branch and referred to as “probability of branch selection”. However, at some levels, there are branches that do not depend on the status of the partner’s previous decoding process, indicated by the absence of $P(OK_{12}^{(j)})$ or $P(\overline{OK}_{12}^{(j)})$. Each node of the tree is labeled by $X(y, z)$ where the capital letter $X$ is the node name while $z$ represents the estimated total number of retransmission frames and $y$ indicates the maximum diversity order that the destination can obtain at the node.
5.3.2 Probability of Branch Selection

As indicated in the previous subsection, three pairs of branch selection probabilities that need to be computed are \( \{P(OK_{12}^{(1)}), P(OK_{12}^{(1)})\} \), \( \{P(OK_{12}^{(2)}), P(OK_{12}^{(2)})\} \) and \( \{P(OK_{12}^{(3)}), P(OK_{12}^{(3)})\} \).

First, \( P(OK_{12}^{(1)}) \) is the probability that User 2 fails to decode User 1’s information after the User 1’s first transmission attempt. Assuming binary phase shift keying (BPSK) modulation and coherent demodulation, one has

\[
P(OK_{12}^{(1)}) = 1 - \int_{0}^{\infty} \left[ 1 - Q\left( \sqrt{2\xi_{12}^{(1)}} \right) \right]^{K} f(\xi_{12}^{(1)}) d\xi_{12}^{(1)} \tag{5.1}
\]

where \( K \) is the length of the information frame, \( \xi_{12}^{(j)} \) is the instantaneous signal-to-noise ratio between User 1 (source) and User 2 (partner) at the \( j \)th transmission attempt (here \( j = 1 \)). As mentioned before, it is assumed that the channel gain is constant for the whole coded frame but it changes independently from one frame to the next. This means that \( \xi_{12}^{(j)} \) are independent random variables. Also since this work focuses on the Rayleigh fading channels, \( \xi_{12}^{(j)} \) have the exponential distribution denoted by \( f(\cdot) \) as expressed in Equation (2.9).

Second, \( P(OK_{12}^{(2)}) \) is the probability that User 2 fails to decode User 1’s information after User 1 conducts the second transmission attempt. This probability is derived in [44] and bounded as

\[
P(OK_{12}^{(2)}) \leq P(D_{12}^{(2)}) \tag{5.2}
\]

where \( D_{12}^{(j)} \) is the event that the errors are detected at User 2 when the decoder operates on the convolutional code with rate \( 1/j \). The quantity \( P(D_{12}^{(2)}) \) can be computed as follows [38], [39]:

\[
P(D_{12}^{(2)}) = 1 - \int_{0}^{\infty} \int_{0}^{\infty} (1 - p_{2})^{K} f(\xi_{12}^{(1)}, \xi_{12}^{(2)}) d\xi_{12}^{(1)} d\xi_{12}^{(2)} \\
\approx \int_{0}^{\infty} \int_{0}^{\infty} (Kp_{2}) f(\xi_{12}^{(1)}, \xi_{12}^{(2)}) d\xi_{12}^{(1)} d\xi_{12}^{(2)} \tag{5.3}
\]
where
\[
p_2 = \min \left[ 1, \sum_{d=d_{\text{free}}^{(2)}}^{\infty} A_d^{(2)} Q \left( \sqrt{2d_1 \xi_{12}^{(1)}} + 2d_2 \xi_{12}^{(2)} \right) \right]
\] (5.4)

In (5.4), \(A_d^{(j)}\) is the number of error events with Hamming weight \(d\), \(d_{\text{free}}^{(j)}\) is the free Hamming distance of the rate-1/\(j\) convolutional code (here \(j = 2\)), \(d_1\) and \(d_2\) are the Hamming weights of the transmission frames \(C_1(x)\) and \(C_2(x)\), respectively, and \(d = d_1 + d_2\). The values of \(d_1\) and \(d_2\) can be approximated according to \(d_1/d_2 = N_1/N_2\) where \(N_1\) and \(N_2\) are the frame length of \(C_1(x)\) and \(C_2(x)\), respectively [37]. The results in this chapter and in [37] demonstrates that this assumption is reasonable.

Finally, \(P(\overline{OK_{12}^{(3)}})\) is the probability that User 2 fails to decode User 1’s information after User 1’s third transmission attempt. This probability can be bounded as follows [38], [39]:
\[
P(\overline{OK_{12}^{(3)}}) \leq P(D_{12}^{(3)})
\] (5.5)

Similar to \(P(D_{12}^{(2)})\), \(P(D_{12}^{(3)})\) is computed as [44]:
\[
P(D_{12}^{(3)}) = 1 - \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} (1 - p_3)^K f(\xi_{12}^{(1)}, \xi_{12}^{(2)}, \xi_{12}^{(3)}) d\xi_{12}^{(1)} d\xi_{12}^{(2)} d\xi_{12}^{(3)}
\]
\[
\approx \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} (Kp_3) f(\xi_{12}^{(1)}, \xi_{12}^{(2)}, \xi_{12}^{(3)}) d\xi_{12}^{(1)} d\xi_{12}^{(2)} d\xi_{12}^{(3)}
\] (5.6)

where
\[
p_3 = \min \left[ 1, \sum_{d=d_{\text{free}}^{(3)}}^{\infty} A_d^{(3)} Q \left( \sqrt{2d_1 \xi_{12}^{(1)}} + 2d_2 \xi_{12}^{(2)} + 2d_3 \xi_{12}^{(3)} \right) \right]
\] (5.7)

and \(d_1\), \(d_2\) and \(d_3\) are the Hamming weights of the frames sent in the first, second and third transmission attempts, respectively, with \(d \approx d_1 + d_2 + d_3\).

Since \(\overline{OK_{12}^{(j)}}\) is the compliment of the event \(\overline{OK_{12}^{(j)}}\); it follows that
\[
P(\overline{OK_{12}^{(j)}}) = 1 - P(\overline{OK_{12}^{(j)}}), \quad j = 1, 2, 3.
\] (5.8)

### 5.3.3 Throughput

For convenience and without loss of generality, the normalized number of transmission and retransmission packets is used throughout this chapter. Therefore, consider
the transmission process for 1 information packet. In the first attempt, User 1 trans-
mits its first frame, \( C_1(x) \), to the destination. This transmission process is presented
by the branch from the root to node \( A \) on the tree in Fig. 5.3. Let \( P(R_{d,j}) \) be
the block error probability, where \( R_{d,j} \) denotes the event that the received frame has
errors at the \( j \)th transmission attempt. As can be seen from the description of the
HARQ protocol, any frame which has error(s) at the destination must be retransmit-
ted. Hence, the normalized estimated number of retransmission packets at node \( A \) is
\[
N_A = P(R_{d,1})
\]
(5.9)

The probability \( P(R_{d,1}) \) can be computed as follows:
\[
P(R_{d,1}) = 1 - \int_0^\infty \left[ 1 - Q\left(\sqrt{2\xi_{10}^{(1)}}\right)\right]^K f(\xi_{10}^{(1)}) d\xi_{10}^{(1)}
\]
(5.10)
where \( \xi_{10}^{(j)} \) is the instantaneous SNR of the channel between User 1 (source) and User
0 (destination) at the \( j \)th transmission attempt (here \( j = 1 \)).

In the second attempt, there are the following two possible retransmission modes:
cooperative (indicated by the solid branch \( AB \)) with probability \( P(OK_{12}^{(1)}) \) and non-
cooperative (indicated by the dashed branch \( AC \)) with probability \( P(\overline{OK}_{12}^{(1)}) \). There-
fore, at node \( B \), the maximum attainable diversity order is 3 and the estimated
number of retransmission frames is
\[
N_B = N_A P(OK_{12}^{(1)}) P(D_{B,2}^{(2)}, R_{d,2}|R_{d,1})
\]
\[
= P(D_{B,2}^{(2)}, R_{d,2}, R_{d,1}) P(OK_{12}^{(1)})
\]
\[
\leq P(D_{B,2}^{(2)}) P(OK_{12}^{(1)})
\]
(5.11)
where \( D_{X,n}^{(j)} \) denotes the event that errors are detected at the destination after the
\( j \)th transmission attempt at node \( X \) with the obtained diversity order of \( n \). The
probability \( P(D_{X,n}^{(j)}) \) is computed as \[38], \[39]:
\[
P(D_{X,n}^{(j)}) = 1 - \int_0^\infty \cdots \int_0^\infty (1-p_n)^K f(\alpha_1, \cdots, \alpha_n) d\alpha_1 \cdots d\alpha_n
\]
\[
\approx \int_0^\infty \cdots \int_0^\infty (Kp_n)f(\alpha_1, \cdots, \alpha_n) d\alpha_1 \cdots d\alpha_n
\]
(5.12)
where

\[
p_n = \min \left[ 1, \sum_{d=d_{\text{free}}}^{\infty} A_d^{(j)} Q \left( \sqrt{2d_1 \alpha_1 + \cdots + 2d_n \alpha_n} \right) \right], \quad (5.13)
\]

In (5.13), \( A_d^{(j)} \) is the number of error events with Hamming weight \( d = d_1 + \cdots + d_n \) of the rate-1/\( j \) code, \( \alpha_i, i = 1, 2, \cdots, n \), is the instantaneous SNR of the underlying channel, which can either be \( \xi_{10}^{(j)} \) or \( \xi_{20}^{(j)} \), and \( \xi_{20}^{(j)} \) is the instantaneous SNR of the channel between User 2 (partner) and User 0 (destination) at the \( j \)th transmission attempt, where \( j = 1, 2, \cdots, L \).

The estimated number of retransmission frames on branch AC is

\[
N_A - N_A P(OK_{12}^{(1)}) = N_A P(OK_{12}^{(1)}) \quad (5.12)
\]

The estimated number of retransmission frames at node C is

\[
N_C = N_A P(OK_{12}^{(1)}) P(D_{C,2}^{(2)} | R_{d,2}^{(1)})
\]

\[
= P(D_{C,2}^{(2)} | R_{d,2}^{(1)}) P(OK_{12}^{(1)})
\]

\[
\leq P(D_{C,2}^{(2)}) P(OK_{12}^{(1)}) \quad (5.14)
\]

where \( P(D_{C,2}^{(2)}) \) is computed from (5.12) with \( n = 2 \) and \( j = 2 \).

In the third attempt, there are three possible branches BD, CE and CF. At node D, the destination decodes User 1’s information with the rate-1/3 code, and the maximum diversity order is 5. Thus the estimated number of retransmission frames at node D is

\[
N_D = N_B P(D_{D,5}^{(3)} | R_{d,3}^{(2)} | D_{B,3}^{(2)} | R_{d,2}^{(1)})
\]

\[
= P(D_{D,5}^{(3)} | R_{d,3}^{(2)} | D_{B,3}^{(2)} | R_{d,2}^{(1)}) P(OK_{12}^{(1)})
\]

\[
\leq P(D_{D,5}^{(3)}) P(OK_{12}^{(1)}) \quad (5.15)
\]

The tree proceeds to node E if User 2 fails to decode User 1’s information after the first transmission attempt, but succeeds after the second attempt. Therefore, the destination decodes the information for User 1 with the rate-1/3 code and the
maximum diversity order is 4. The estimated number of retransmission frames at node $E$ is

$$N_E = N_C P(OK_{12}^{(2)}) P(D_{E,4}^{(3)}, R_{d,3}, D_{C,2}^{(2)}, R_{d,2}, R_{d,1})$$

$$= P(D_{E,4}^{(3)}, R_{d,3}, D_{C,2}^{(2)}, R_{d,2}, R_{d,1}) P(OK_{12}^{(2)}) P(OK_{12}^{(1)})$$

$$\leq P(D_{E,4}^{(3)}) P(OK_{12}^{(2)}) P(OK_{12}^{(1)})$$  (5.16)

Node $F$ is selected when User 2 fails to decode User 1’s information after two transmission attempts. The decoder operates on the rate-1/3 code, and the maximum diversity order is 3. The estimated number of retransmission frames in this case is:

$$N_F = N_C P(OK_{12}^{(2)}) P(D_{F,3}^{(3)}, R_{d,3}, D_{C,2}^{(2)}, R_{d,2}, R_{d,1})$$

$$= P(D_{F,3}^{(3)}, R_{d,3}, D_{C,2}^{(2)}, R_{d,2}, R_{d,1}) P(OK_{12}^{(2)}) P(OK_{12}^{(1)})$$

$$\leq P(D_{F,3}^{(3)}) P(OK_{12}^{(2)}) P(OK_{12}^{(1)})$$  (5.17)

With $T = 4$, the process stops transmitting after four transmission attempts. Therefore, the estimated total number of frames delivered by User 1 and User 2 to the destination for one User 1’s information frame is

$$N_{total} = 1 + N_A + N_B + N_C + N_D + N_E + N_F$$

$$= 1 + P(R_{d,1}) + P(D_{B,3}^{(2)}) P(OK_{12}^{(1)}) + P(D_{C,2}^{(2)}) P(OK_{12}^{(1)})$$

$$+ P(D_{D,5}^{(3)}) P(OK_{12}^{(1)}) + P(D_{E,4}^{(3)}) P(OK_{12}^{(2)}) P(OK_{12}^{(1)})$$

$$+ P(D_{F,3}^{(3)}) P(OK_{12}^{(2)}) P(OK_{12}^{(1)})$$  (5.18)

Finally, the throughput of the proposed HARQ scheme for coded cooperative communications can be determined as

$$\eta_{coop} = \frac{1}{N_{total}} \frac{K - \nu}{K + m} \approx \frac{1}{N_{total}}$$  (5.19)

where, recall that, $K$ is the length of the input frame of the convolutional code (including the CRC bits), $m$ is the number of termination bits and $\nu$ is the number of CRC bits. A common practice is to ignore $\nu$ (i.e., set $\nu = 0$) in the above expression in approximating the throughput.
5.3.4 End-to-End Bit Error Probability

The tree structure in Fig. 5.3 can terminate at one of the four nodes $G$, $H$, $I$, $J$. In general, which node the tree ends at depends on the inter-user channel quality.

The tree terminates at node $G$ corresponding to the route [root $\to$ $A$ $\to$ $B$ $\to$ $D$ $\to$ $G$]. This route depends on User 2’s first decoding attempt quantified by $P(OK_{12}^{(1)})$ as in (5.1) and (5.8). If this node is the termination node, the maximum diversity order of 7 can be obtained. Hence, the bit error probability at node $G$ is [38], [39]:

$$P_{b,G} = \int_0^\infty \cdots \int_0^\infty \min \left[ \frac{1}{2}, p_G \right] f(\alpha_1, \cdots, \alpha_7) d\alpha_1 \cdots d\alpha_7 \quad (5.20)$$

where

$$p_G = \frac{1}{k_c} \sum_{d=d_{free}^{(4)}} \infty C_d^{(4)} Q \left( \sqrt{2d_1\alpha_1 + \cdots + 2d_7\alpha_7} \right) \quad (5.21)$$

and $k_c$ is the number of input bits per branch of the code trellis. The parameter $C_d^{(4)}$ is the number of information bits in error, corresponding to error events with Hamming weight $d = d_1 + \cdots + d_7$.

The tree terminates at node $H$ via route [root $\to$ $A$ $\to$ $C$ $\to$ $E$ $\to$ $H$]. The probability of following this route is $P(OK_{12}^{(1)}) \cdot P(OK_{12}^{(2)})$, computed based on (5.1), (5.8) and (5.2). This means that User 2 fails to decode User 1’s information in the first attempt but does succeed in the second one. The diversity order of 6 is obtained if the tree ends at this node. The bit error probability is [38], [39]:

$$P_{b,H} = \int_0^\infty \cdots \int_0^\infty \min \left[ \frac{1}{2}, p_H \right] f(\alpha_1, \cdots, \alpha_6) d\alpha_1 \cdots d\alpha_6 \quad (5.22)$$

where

$$p_H = \frac{1}{k_c} \sum_{d=d_{free}^{(4)}} \infty C_d^{(4)} Q \left( \sqrt{2d_1\alpha_1 + \cdots + 2d_6\alpha_6} \right) \quad (5.23)$$

and $d = d_1 + \cdots + d_6$.

The tree terminates at node $I$ via the route [root $\to$ $A$ $\to$ $C$ $\to$ $F$ $\to$ $I$]. The probability of this route is $P(OK_{12}^{(1)}) \cdot P(OK_{12}^{(2)}) \cdot P(OK_{12}^{(3)})$, computed from (5.1), (5.2), (5.5) and (5.8). The maximum diversity order of 5 is obtained if the tree ends
at node $I$. Therefore, the bit error probability is expressed as [38], [39]:

$$P_{b,I} = \int_0^\infty \cdots \int_0^\infty \min \left[ \frac{1}{2}, p_I \right] f(\alpha_1, \cdots, \alpha_5) d\alpha_1 \cdots d\alpha_5 \quad (5.24)$$

where

$$p_I = \frac{1}{k_c} \sum_{d=d_{free}^{(4)}}^\infty C_d^{(4)} Q\left(\sqrt{2d_1\alpha_1 + \cdots + 2d_5\alpha_5}\right) \quad (5.25)$$

and $d = d_1 + \cdots + d_5$.

Finally, the worst case is that the tree terminates at node $J$. The route is [root $\rightarrow$ A $\rightarrow$ C $\rightarrow$ F $\rightarrow$ J], which happens with probability $P(OK^{(1)}_{12}) \cdot P(OK^{(2)}_{12}) \cdot P(OK^{(3)}_{12})$ and can be calculated from (5.1), (5.2) and (5.5). This possibility happens when User 2 does not succeed in any of the three decoding attempts. The bit error probability at this node is, [38], [39]:

$$P_{b,J} = \int_0^\infty \cdots \int_0^\infty \min \left[ \frac{1}{2}, p_J \right] f(\alpha_1, \cdots, \alpha_4) d\alpha_1 \cdots d\alpha_4 \quad (5.26)$$

where

$$p_J = \frac{1}{k_c} \sum_{d=d_{free}^{(4)}}^\infty C_d^{(4)} Q\left(\sqrt{2d_1\alpha_1 + \cdots + 2d_4\alpha_4}\right) \quad (5.27)$$

and $d = d_1 + \cdots + d_4$.

Based on Equations (5.20), (5.22), (5.24) and (5.26), the average bit error rate expression for the cooperative coded system with the proposed HARQ protocol can be determined as:

$$P_b = P(OK^{(1)}_{12})P_{b,G} + P(OK^{(1)}_{12})P(OK^{(2)}_{12})P_{b,H} + P(OK^{(1)}_{12})P(OK^{(3)}_{12})P_{b,I} + P(OK^{(1)}_{12})P(OK^{(2)}_{12})P(OK^{(3)}_{12})P_{b,J} \quad (5.28)$$

5.4 Illustrative Results and Discussions

As an illustrative example, we investigate in this section the throughput and BER performance of a simple 3-node cooperation topology. For convenience, the BPSK modulation scheme is used and thus the average SNRs, i.e., $\xi_{12}$, $\xi_{10}$, and $\xi_{20}$, collapse to the $E_b^{(1,2)}/N_0$, $E_b^{(1,0)}/N_0$ and $E_b^{(2,0)}/N_0$ respectively, where $E_b$ is the average bit
energy, $N_0$ is the one sided power spectral density of white Gaussian noise, and the subscript $(i, j)$ denotes the channel from source $i$ to the destination $j$. Therefore, $E_b^{(i,j)}/N_0$ is used in all graphic representations in this section. The convolutional code employed has rate $1/2$ and its generator polynomials are $g_1 = 5$ and $g_2 = 7$ in octal form. The information frame length is set to $K = 128$ bits, including the CRC bits. Both simulations and numerical computations based on the analytical framework presented in Section 5.3 are carried out. Multilayer integrals in all the equations in this chapter are calculated by the Gauss-Laguerre approximation method [36]. In addition, the limit-before-average technique [38], [39] is used to obtain the tight bounds. All illustrative results here are for User 1. Due to the symmetry, similar analysis can be carried out and analogous results can be obtained for User 2.

First, the throughput of the proposed HARQ protocol is considered. Simulation and numerical results are presented in Fig. 5.4. One can see that the analysis and simulation curves are very close to each other. This verifies that the through-
put expressions have been properly developed. Moreover, Fig. 5.4 shows that the throughput depends very little on the inter-user channel quality. It also does not change significantly when comparing between the non-cooperative and cooperative systems.

The above observation can be explained as follows. Consider the frame error rate (FER) at each transmission level shown in Fig. 5.5. One can see that the FER of node A mainly contributes to the summation in (5.18). However, this node is the common node for all cases of the inter-user channel qualities. Therefore, the dominant FER of this node does not make the throughput of the cooperative system differ from that of the non-cooperative system. After this transmission level, at level 2, there is a slight difference between the FER of node B and that of node C, where node B corresponds to the cooperative transmission and node C corresponds to the non-cooperative transmission, respectively. Hence, these two nodes do not make any significant change in the summation in (5.18). Finally, consider the last three nodes, D, E and F, that must be taken into account for $N_{total}$ computation. One

**Figure 5.5** Frame error probabilities at different node levels.
can see that the FER curves for nodes $F$ and $E$ are very close and the explanation is the same as for the pair of nodes $B$ and $C$ above. Although there is a big gap between the FER curve of node $D$ and those of nodes $E$ and $F$, the effect of this gap on the summation in (5.18) is shadowed by the dominant FER of node $A$ as seen from Fig. 5.5. In summary, the reason for the similar throughput between the non-cooperative and cooperative systems is that they have the same dominant contributing component on their average throughput computed as in (5.18). It was also noted that, in the low range of SNR, the system throughput of the proposed HARQ protocol quickly increases compared to that of the HARQ reported in [50]. At a high SNR, the approximated throughput (including the CRC bits) of the proposed protocol approaches $\frac{K-v}{K} \approx 1$ bit/s/Hz whereas this parameter is only 0.65 bits/s/Hz for the HARQ in [50].

Now, consider the bit error performance of the proposed HARQ protocol. First, Fig. 5.6 presents the bit error rate (BER) versus the signal to noise ratio of User 1.
at the destination, i.e., $E_b^{(1,0)}/N_0$. Both the simulation and analytical results confirm the accuracy of the bit-error-probability calculation developed in the previous section. By employing the proposed HARQ at both the partner and the destination, the error performance of the proposed protocol can approach that of the perfect inter-user channel at the signal to noise ratio level of $E_b^{(1,2)}/N_0 \approx 15$ dB as shown in Fig. 5.6.

It is interesting to observe that with the same delay as in the HARQ protocol of a non-cooperative system, the proposed HARQ protocol in the cooperative system offers approximately from 2.0 dB up to 3.5 dB gain at the required BER level of $10^{-5}$ depending on the inter-user channel quality. At a lower BER level, i.e., a better target performance, this gain is increased (about 4.0 dB). These substantial gains clearly indicate the advantages in using cooperative coding in conjunction with the HARQ protocol to improve the system performance and power efficiency by exploiting both time diversity and spatial diversity.

5.5 Summary

A power-efficient combined cooperative coding and HARQ was proposed in this chapter. Accurate analytical expressions were developed to evaluate both the throughput and end-to-end bit error performance of the proposed scheme. Simulation and analytical results show that while the throughput of the system is very robust to inter-user channel qualities, the end-to-end bit-error-rate (BER) is significantly improved for any practical inter-user channel quality. The proposed scheme offers a significant power saving, especially at high link performance requirements. Those advantages of the proposed scheme make it a good candidate for some wireless applications which require high link reliability and are insensitive to delay such as data transmission in cellular networks and transmission of control and surveillance data over wireless sensor networks [51].
6. Conclusions and Suggestions for Further Studies

6.1 Conclusions

This thesis is concerned with efficient and reliable transmission over wireless channels by means of cooperative diversity. Two main contributions have been made in this thesis. The first contribution is a proposal of a bandwidth-efficient coded cooperative system in which high-order modulation schemes such as \( M\)-QAM or \( M\)-PSK are used not only to increase the transmission rate but also to provide a means to improve the error performance over block fading channels. Both performance analysis and simulation results demonstrated that the power gain of the proposed system over the conventional system is about 2.0 dB. In addition, by adjusting the repetition level, a new and effective adaptive transmission is obtained to increase the system throughput.

The second contribution of this thesis is a novel transmission protocol based on H-ARQ for cooperative coded systems to further reduce the negative impact of block fading on the error performance. The proposed transmission protocol combines temporal diversity and spatial diversity to deliver a higher diversity order. The benefits of the proposed transmission protocol were verified by both analysis and simulation. More specifically, power gain as high as 3.5 dB can be obtained by the proposed transmission protocol. This significant gain makes the proposed transmission protocol suitable for many wireless networks where the link reliability is the major concern.
6.2 Suggestions for Further Studies

The transmission protocols designed in this thesis are for the “static” cooperation situation. This means that the users do not change their partners during at least one activation period. The drawback of static cooperation is that it loses the chance to obtain full diversity, i.e., order of 2 for pairwise cooperation. This is because when the link from a certain partner to the destination falls in the deep fading state, the considered user is unlikely to obtain the full diversity order. Instead, “dynamic” cooperation can be considered. That is, during one activation period, the partners for a given user can be changed according to the partner-destination channel state information. This is somewhat similar to the handoff or handover process in cellular networks. By doing this, it is likely that the considered user can obtain the full diversity gain almost all the time. As a result, the performance is further improved with the cost of added complexity. Dynamic cooperation is practical for wireless networks in which the wireless terminals move at a very low speed such as the pedestrians in cellular networks or moving sensors in wireless sensor networks.

It is seen from Fig. 4.7 (Chapter 4) that the performance of the proposed bandwidth-efficient coded cooperation scheme is poorer than that of the conventional scheme when the average signal to noise ratio of the inter-user channel is less than 20 dB, i.e., the inter-user channel is far from being perfect. This disadvantage can be reduced or eliminated by applying distributed turbo coding with soft information techniques [52], which has a better capability to remove the effect of imperfect inter-user channels. In particular, with this technique, the transmitted information is not decoded by the partner. Instead, the partner calculates and forwards the corresponding soft information in a similar fashion investigated in [52].
A. Computing $P(\Theta)$

In order to obtain the probability distribution of the random variable $\Theta$, the block error rate (BLER) is required. As discussed in [37], [39], [42], the conditional BLER can be bounded as

$$P_{\text{block}}(\xi) \leq 1 - [1 - P_E(\xi)]^K \leq KP_E(\xi) \quad (A.1)$$

where $K$ is the number of trellis branches in the codeword, $P_E(\xi)$ is the error event probability and $\xi$ is the instantaneous interuser channel SNR. The quantity $P_E(\xi)$ is upper bounded by

$$P_E(\xi) \leq \sum_{d=d_{\text{free}}}^{\infty} A_d P(d|\xi) \quad (A.2)$$

where $d_{\text{free}}$ denotes the free distance of the trellis code and $A_d$ is the number of the error events with Hamming weight $d$.

Based on (A.1), the conditional probability distribution of $\Theta$ can be determined as follows.

- **Case 1 ($\Theta = 1$):**

  $$P(\Theta = 1|\xi) = (1 - P_{\text{block}}^{(1)}(\xi))(1 - P_{\text{block}}^{(2)}(\xi)) \leq (1 - P_E^{(1)}(\xi))^K (1 - P_E^{(2)}(\xi))^K \leq (1 - KP_E^{(1)}(\xi))(1 - KP_E^{(2)}(\xi)) \quad (A.3)$$

  where $P_E^{(1)}$ and $P_E^{(2)}$ denote the error event probabilities at User 1 and User 2, respectively.
• Case 2 ($\Theta = 2$):

\[
P(\Theta = 2 | \xi) = P_{\text{block}}^{(1)}(\xi)(1 - P_{\text{block}}^{(2)}(\xi))
\leq [1 - (1 - P_E^{(1)}(\xi))^K](1 - P_E^{(2)}(\xi))^K
\leq KP_E^{(1)}(\xi)[1 - KP_E^{(2)}(\xi)]
\]  

(A.4)

• Case 3 ($\Theta = 3$):

\[
P(\Theta = 2 | \xi) = (1 - P_{\text{block}}^{(1)}(\xi))P_{\text{block}}^{(2)}(\xi)
\leq (1 - P_E^{(1)}(\xi))^K[1 - (1 - P_E^{(2)}(\xi))^K]
\leq K[1 - KP_E^{(1)}(\xi)]P_E^{(2)}(\xi)
\]  

(A.5)

• Case 4 ($\Theta = 4$):

\[
P(\Theta = 2 | \xi) = P_{\text{block}}^{(1)}(\xi)P_{\text{block}}^{(2)}(\xi)
\leq [1 - (1 - P_E^{(1)}(\xi))^K][1 - (1 - P_E^{(2)}(\xi))^K]
\leq K^2P_E^{(1)}(\xi)P_E^{(2)}(\xi)
\]  

(A.6)

The unconditional probability distribution of $\Theta$ is then computed as

\[
P(\Theta) = \int_{\xi} P(\Theta | \xi)f(\xi)d\xi
\]  

(A.7)

Finally, it should be noted that the limit-before-average technique in [39] can also be used to obtain the tight bound. For example, computing (bounding) $P(\Theta = 1)$ with the limit-before-average technique yields

\[
P(\Theta = 1) \leq \int_{\xi}(1 - \min[1, P_E^{(1)}(\xi)])^K \times (1 - \min[1, P_E^{(2)}(\xi)])^K f(\xi)d\xi
\]  

(A.8)
References


