Practical Applications of Coding

Invited Paper

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***Abstract*—Coding applications have grown rapidly in the past several years with cost-effective performance demonstrated on several channels. Convolutional coding with soft-decision Viterbi decoding has emerged as a standard technique and is particularly well adapted to the communication satellite channel. Decoder implementations are discussed and examples are cited. Robustness of code performance is emphasized and instances of actual coding gain surpassing theoretical or basic coding gain are given. Some promising future directions are noted.**

A. Introduction

IN THE 25 years since Shannon's seminal papers of 1948 and 1949, coding theory has progressed rather fitfully through periods of euphoric highs with discoveries of promising code classes, elegant decoding algorithms, and visions of revolutionizing communications, and dismal lows when it was feared that coding application would never move beyond hard decision Hamming and Golay codes. It now appears safe to say that coding is maturing into an important segment of communications systems engineering, one that will see increasing applications of relatively standard techniques to reduce system cost and complexity. Revolutions are not yet in sight.

Certain of the reasons for the increasing applications of coding are external: the anticipated rapid growth in satellite communications; the revolution in digital integrated circuits; the increasing emphasis on the reliable transmission of digital data and of digitally coded analog signals; the availability of inexpensive computers for system, algorithm, and hardware simulation; the increasing digitalization of modems, switches, and interconnect facilities, permitting ready interfacing and common maintainability; and the increasing sophistication of the user community.

Equally important are developments within the coding field. Clearly, the inventions of sequential decoding by Wozencraft [1], [2] and its refinement by Fano [3], of threshold decoding by Massey [4], [5], of effective BCH decoding by Berlekamp [6], and of the Viterbi algorithm [7] were all critical to the significant application of coding theory. Two less obvious factors played an important role. The first, given initial impetus by Kohlenberg, Massey, Forney, and Gallager at Codex Corporation [8], was the decision to foresake revolutions and to stress moderate but cost-effective coding gains utilizing basically short convolutional codes. This point of view was given considerable reinforcement by Heller [9], [10] who demonstrated the power of applying the Viterbi algorithm to decode short convolutional codes using soft decisions. I cite this change in emphasis as important, even though it might seem obvious, and refer to it as the introduction of engineering to coding. As evidence that it was not obvious, I note that Viterbi originally presented his algorithm as asymptotically efficient for long codes, and that a full 19 months passed before emphasis was placed on short codes and the practical importance of Viterbi decoding was really recognized.

The second factor was somewhat fortuitous; namely, the verification through analysis, computer simulation, and real channel tests that, when appropriately matched to a system, coding provides robust performance improvements, often in excess of the theoretical coding gain calculated for a Gaussian channel. This ability to perform well on real channels, despite interference, intermodulation and cochannel noise, and multipath, and furthermore to reduce the stress on other system components, by permitting the part of the system between coder and decoder to operate at a higher noise level, has substantially increased the attractiveness of coding. In Section C, we examine these aspects in more detail.

Section B of this report is devoted to a comparison of Viterbi decoding and sequential decoding for white Gaussian noise channels, with most attention directed to the former technique. In my opinion, Viterbi decoding is presently, and will remain for a long period, the most important decoding technique providing coding gain for a wide variety of channels. It does not of course answer all problems, and other coding techniques will continue to have significance in limited applications. Block codes will continue as the preferred technique for providing error detection for variable-length messages and will assume increasing importance in error correction and detection within computers and computer peripherals and memories where the natural organization of the data is in short blocks. Block codes with soft decisions [11], [12] and with burst-trapping properties have also been advocated for various channels and will continue to find some use, but it is my opinion that performance-versus-complexity comparisons and system integration requirements strongly favor convolutional codes.

Hard-decision feedback decoding of convolutional codes, including threshold and diffuse decoding, particularly with internal interleaving and deinterleaving, will continue to find increasing application on fading channels, such as the high frequency (HF) radio channel, on bursty channels such as the telephone channel when detection and repeat request

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TABLE I
BASIC CODING GAIN (dB) FOR SOFT-DECISION VITERBI DECODING AND HARD-DECISION SEQUENTIAL DECODING

		Soft Decision Viterbi Decoding									Hard Decision Sequential Decoding	
E_b/N_o Uncoded (db)	R=	1/3		1/2			2/3		3/4		1/2	
	P_b \ K	7*	8	5	6*	7*	6	8*	6	9*	41* (1)	47* (2)
6.8	10^{-3}	4.2	4.4	3.3	3.5	3.8	2.9	3.1	2.6	2.6	1.5	3.0
9.6	10^{-5}	5.7	5.9	4.3	4.6	5.1	4.2	4.6	3.6	4.2	3.8	4.2
11.3	10^{-7}	6.2	6.5	4.9	5.3	5.8	4.7	5.2	3.9	4.8	4.8	6.5
——	Upper Bound	7.0	7.3	5.4	6.0	7.0	5.2	6.7	4.8	5.7	7.4	7.4

*—Presently implemented.
(1)—Speed factor = 1.75; Buffer = 2^{15} bits.
(2)—Speed factor = 266; Buffer = 2^{16} bits.
The coding gains are relative to the uncoded energy-per-bit-to-noise power density given in the leftmost column.

is inconvenient, and increasingly, as the outer coding technique in concatenated coding schemes. In this latter case, Viterbi decoding might protect all of the traffic on a satellite link, while hard-decision feedback decoding would provide additional protection for critical data end-to-end (including the interconnect facilities as well as the satellite link). Threshold decoders are presently finding extensive application for the protection of satellite links in the Spade system [13].

Finally, sequential decoding is finding limited application in deep-space links [14], some NASA satellite links, and in certain low-data-rate links requiring maximum coding gain. Hard-decision sequential decoding provides improved coding gain over soft-decision Viterbi decoding only at low error probabilities or high speed factors (ratio of computational speed to data rate). Soft-decision sequential decoders do provide performance improvements, but at significant cost. They are also less robust and more difficult to incorporate in systems because of the need for synchronization and the requirement to accommodate output error bursts. In the future, some of these requirements for high coding gain may be better performed by concatenated systems with Reed–Solomon outer codes and Viterbi inner codes [15]–[17].

B. Basic Coding Gain on Gaussian Channels

The basic coding gain achieved by use of a code on white Gaussian channels can be specified in terms of the reduction below ideal binary phase shift keying (BPSK) of the energy-per-bit to noise power density ratio E_b/N_0 required to achieve a given bit error rate when operating on an ideal additive white Gaussian noise channel. We stress the point that E_b is the energy per information bit; thus for a rate-1/2 code, the transmitted symbol energy E_s is 3 dB less than E_b.

The basic coding gains of a number of codes are presented in Table I for bit error probabilities of 10^{-3}, 10^{-5}, and 10^{-7}. All coding gains are relative to the value of E_b/N_0 shown in the leftmost column required by uncoded BPSK to achieve the specified bit error probability. Codes with asterisks have presently been implemented.

As expected, coding gain increases as the required bit error probability is decreased. Coding gain cannot increase indefinitely, however, and Table I also contains a simple upper bound. This bound is obtained by noting that, for a code with rate R and minimum free distance d

$$P_b \geq Q(\sqrt{2E_bRd/N_0})$$

where $Q(x)$ is the integral of the tail beyond x of the Gaussian density function, while, for uncoded BPSK [23]

$$P_{b,\,\text{uncoded}} = Q(\sqrt{2E_b/N_0}).$$

Thus

$$\text{coding gain} \leq Rd.$$

For example, the rate-1/2 constraint-length-7 code has $d = 10$ and hence a maximum coding gain of 5, or 7 dB. The upper bounds include some surprises; in particular, for constraint length 7, the rate-1/2 and -1/3 codes have the same upper bound. Note that the weaker codes tend to be closer to the bound at $P_b = 10^{-7}$ than the more powerful codes; often the upper bound is not tight for any E_b/N_0 of interest.

The reader is directed to the extensive literature on the Viterbi algorithm [7, pp. 18–22] for a detailed discussion of its theory and implementation. The Viterbi decoding schemes whose performance is summarized in Table I all involve binary convolutional codes of relatively short constraint length K, and soft-decision demodulation in which the output is octal, i.e., the integers $0,1,\cdots,7$, where 0 would mean a reliable zero, 3 a questionable zero, 4 a questionable one, and so forth.

As an indication of speed-complexity tradeoffs, we note that 100 Kbit/s and 10 Mbit/s full-duplex soft-decision $K = 7$ rate-1/2 Viterbi decoders have been implemented with 84 and 270 TTL integrated circuits, respectively, and with the high-speed decoder utilizing a custom LSI circuit to perform the central iterative operation. Rate-2/3 and -3/4 decoders over the same speed ranges require approximately 10 percent more IC's as implemented. In each case, speed increases of a factor of approximately three for the same parts count can be achieved by using ECL 10K logic. It also appears feasible to build a 50 Mbit/s rate-2/3 $K = 6$ decoder using ECL logic with moderate parts count. Higher speeds are most effectively obtained by paralleling decoders, with a side benefit that the resulting data is interleaved.

The performance of hard-decision rate-1/2 $K = 41$ and $K = 47$ systematic code sequential decoders with very different speed factors are included for comparison. The upper bound in this case has been decreased by 3 dB to account for the loss due to hard decisions at high E_b/N_0. (No loss has been included for soft decisions.) It appears that use of longer constraint lengths and/or nonsystematic codes and/or soft decisions is necessary to make sequential decoding competitive. A coding gain limit for sequential decoding is also set by the computational variability and is approximately

$$CG \leq \left. \frac{\alpha \ln \mu\Gamma}{E_b/N_0} \right|_{\text{dB}}$$

where α is the Pareto exponent, μ is the decoder speed factor, and Γ is the buffer size [24], [25]. For the very noisy channel

$$\alpha = \frac{E_b}{E_{b,\min}} - 1$$

and the bound can be written

$$CG \leq \ln \mu\Gamma|_{\text{dB}} + 1.6 \text{ dB}.$$

For $\mu\Gamma = 2 \times 10^4$, a reasonable value, this bound becomes 11.5 dB and thus is not the limiting factor for the systematic rate-1/2 code. (However, further refinement of this bound might reduce it to a more critical value.) Sequential decoders of this type have been implemented with TTL logic for data rates up to 5 Mbit/s [26], [27], and with ECL logic for data rates up to 40 Mbit/s [28]. A generalized soft-decision decoder has also been implemented [29].

The lesson of Table I is that fairly simple codes provide significant basic coding gains, even for the higher rate codes. Thus present coding applications are based on these codes, even though they fall well short of Shannon's lower bound on E_b/N_0 for the white Gaussian noise channel of -1.6 dB.

C. Performance on Real Channels

As has been often noted by skeptics, real channels are not additive white Gaussian noise channels and are never fully coherent. Thus actual coding gains can be expected to differ from the basic coding gains cited in Table I. What is surprising is that, in practice, actual coding gains often surpass basic coding gains by significant amounts. We now examine a number of the contributing factors.

First of all, as noted in [22], coding exhibits a surprising robustness of performance in the face of errors in phase tracking, time tracking, and gain tracking. The impact on performance of phase and time errors is about the same for coded as for uncoded BPSK and QPSK in the ranges of interest. However, in the coded case, the loops must be narrower with respect to the bit rate. Thus, for example, to achieve a loop signal-to-noise ratio of 18 dB, the loop filter in an uncoded system operating at $E_b/N_0 = 10$ dB must integrate over at least 6 bits ($10 \log 6 \approx 8$ dB) while the loop filter in a coded system with E_b/N_0 of 5 dB must integrate over at least 20 bits (13 dB) to achieve the same signal-to-noise ratio. Furthermore, when tracking modulated BPSK data with a squaring or Costas loop, losses introduced by the squaring must be overcome by a longer integration time. This increase is proportional to [30]

$$\left(1 + \frac{1}{2E_s/N_0}\right)$$

and remains small, less than 50 percent, as long as the symbol signal-to-noise ratio $E_s/N_0 = RE_b/N_0$ exceeds 0 dB. For a rate-1/2 code with $E_b/N_0 = 4.5$ dB, $E_s/N_0 = 1.5$ dB. (It should be noted that poor loop design can increase this factor somewhat.)

Despite the use of soft decisions, the coding gain is relatively insensitive to automatic gain control (AGC) errors. Errors of ± 3 dB decrease coding gain by 0.1 dB or less [22].

When tracking phase on data-modulated signals, 180° phase inversions can occur. If the all-ones word is a codeword, the code is said to be transparent to phase inversions and differential encoding and decoding can be placed before the encoder and after the decoder, respectively, effectively removing the phase inversion [22], [27]. In the uncoded case, differential decoding doubles the error rate or degrades performance by about 0.3 dB. With coding, errors tend to occur in short bursts and differential decoding typically increases error rate by about 20 percent or less than 0.1 dB.

Real channels also depart from the additive white Gaussian noise channel in that modems and repeaters are nonideal and channels often have multipath, interference, and memory.

A typical modem performance is illustrated in Fig. 1. Curve 1 depicts ideal BPSK or QPSK. Curve 2 illustrates performance achieved by a modem at the desired information bit rate, while curve 3 depicts performance at the higher symbol rate used in a coded system. (Of course, if bandwidth is very tight, curve 3 might be further to the right.) Curves 2 and 3 encompass filter losses, tracking error losses, and other nonidealities present in the modems. Usually these losses cause greatest departure from ideal at very low signal-to-noise ratios (loops lose lock) and very high signal-to-noise ratios (internal noise sources become significant in comparison to the comparatively small front-end additive noise). Ideally, the uncoded system achieves a desired bit rate, say 10^{-5}, at $E_b/N_0 = a$, as in Fig. 1, but in practice actually requires $E_b/N_0 = b$, incurring a loss of L_u dB from ideal, as shown in Fig. 1. Ideally, the coded system with a basic coding gain of G operates at $E_b/N_0 = d$ or, for a rate-R code, $E_s/N_0 = c$, as shown in the figure. The modem,

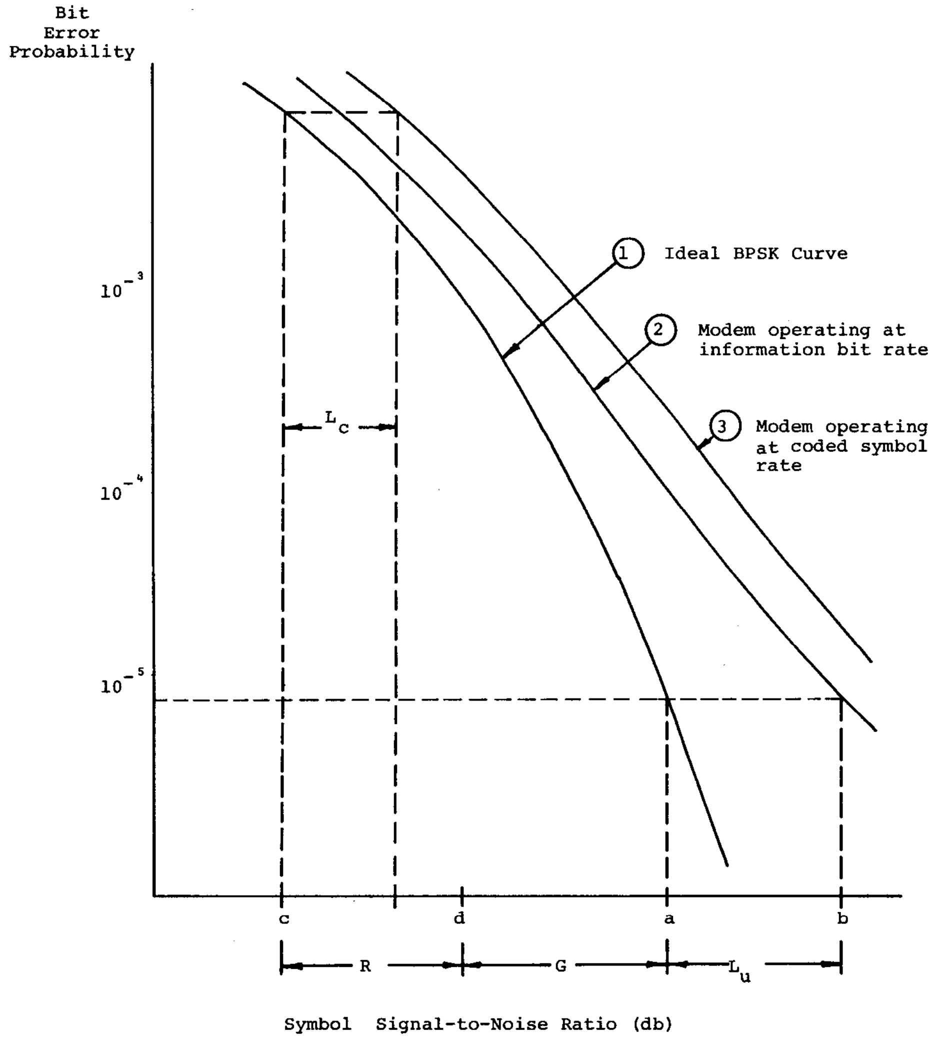

Fig. 1. Ideal and typical PSK modem performance at data rate and code symbol rate. Dashed lines indicate calculation of actual coding gain $L_u + G - L_c$ from ideal coding gain G.

however, suffers a degradation of L_c dB in achieving this operating point. The actual coding gain is thus $G + L_u - L_c$, and often exceeds the basic coding gain G by 1 dB or more.

Additional degradations in uncoded and coded systems arise from noise sources other than receiver noise and from signal-dependent noises. Thus the actual noise-to-signal ratio might be

$$\frac{N_0'}{E_b} = \frac{N_0 + N_1 + xE_b}{E_b} = \frac{N_0}{E_b} + \frac{N_1}{E_b} + x$$

where N_1 is the other signal-independent noise and xE_b represents signal-dependent noise. Suppose $N_1/E_b + x$ is equal to N_0/E_b in the uncoded case. The actual E_b/N_0' is then 3 dB poorer than the ideal. With coding, N_0/E_b may be perhaps 5 dB larger than without coding. If so, $N_1/E_b + x$ is only about 1/3 as large as N_0/E_b and the actual E_b/N_0' with coding is only 1.2 dB poorer than the basic coding gain.

In the previous discussion, we ignored the fact that the actual noise is non-Gaussian, raising questions as to the effectiveness of the soft decisions. The actual impact is minimal, since with coding, the eight decision regions typically extend only from $-3/2\sigma$ to $+3/2\sigma$ and do not rely on small-probability-density tails. Moreover, the Viterbi decoding algorithm is not highly sensitive to the actual probability distribution on the eight output levels. Finally, the effects of large interference can often be somewhat suppressed by the saturation of the output at its highest or lowest level.

In a multipath situation, the improvement in actual coding gain over basic coding gain remains valid even when bandspreading is used to combat the multipath, despite the fact that part of the bandspreading bandwidth must be used for coding redundancy. Consider, for example, a case in which pseudo-noise bandspreading is used to combat a single multipath of strength α relative to the direct signal and the delay between the two paths exceeds one chip of the pseudo-noise code. With n pseudo-noise chips per information bit (bandspreading by a factor of n), with an energy-per-bit-to-noise ratio of E_b/N_0 on the direct path, and with

TABLE II
ACTUAL AND SIMULATED CODING GAINS FOR SINGLE MULTIPATH WITH BIT ERROR PROBABILITY P_b WITH $\alpha = \frac{1}{2}$ AND 1, AND $n = 4, 8, 16, 32, 64$

$\frac{\alpha^2}{n}$	P_b	Actual Coding Gain Predicted From (3.1)	Actual Coding Gain Determined From Simulation	Basic Coding Gain
1/8	8.5×10^{-3}	5.4 db	5.4 db	2.5 db
1/16	2×10^{-3}	5.1 db	4.9 db	3.4 db
1/32	1×10^{-3}	4.7 db	4.6 db	3.8 db

the pseudo-noise correlator in the receiver correctly tracking the delay in the direct path, the observed value of energy-per-bit-to-noise, say E_b/N_0', is approximately

$$2E_b/N_0' = \frac{E_b}{\frac{N_0}{2} + \frac{\alpha^2}{n} E_b} = \frac{2E_b}{N_0}\left(1 + \frac{\alpha^2}{n}\frac{2E_b}{N_0}\right)^{-1}.$$

Here, the noise-like term due to the multipath of energy $\alpha^2 E_b$ is reduced by the bandspreading factor n. The term

$$\left(1 + \frac{\alpha^2}{n}\frac{2E_b}{N_0}\right)^{-1} \quad (1)$$

represents the degradation due to the multipath.

In the coded case, the same calculation may be made for the symbol signal-to-noise ratio, except that the bandspreading factor is reduced to nR, where R is the code rate

$$\frac{2E_b'}{N_0} = \frac{2E_s}{\frac{N_0}{2} + \frac{\alpha^2}{nR} E_s} = \frac{2E_s}{N_0}\left[1 + \frac{\alpha^2}{nR}\frac{2E_s}{N_0}\right]^{-1}.$$

Recognizing that $E_s/R = E_b$, we see that the degradation factor is the same in the coded and uncoded case. However, since E_b/N_0 is smaller with coding, the term $(\alpha^2/n)(2E_b/N_0)$ is small compared to one and the actual degradation is reduced; i.e., the actual coding gain is increased.

To test the validity of (1) for predicting actual coding gain, simulations were made for a multipath channel with two paths separated by two chips and for several values of E_b/N_0, α, and n, without coding and with constraint-length-7 rate-1/2 soft-decision Viterbi decoding. Simulation points representing the same values of α^2/n tended to cluster tightly. Results are presented in Table II in terms of coding gain. The excellent agreement between the simulation results and (1) demonstrates the usefulness of (1) and the significant increases of actual over basic coding gain.

The results also reinforce the observation that the non-Gaussian noise does not greatly affect predicted performance.

D. Future Directions and Application of Coding

In discussing coding applications, I have chosen thus far to concentrate on what has been termed forward error correction (FEC) coding (a useful terminology although error correction is somewhat of a misnomer when soft decisions are used) and particularly on convolutional codes and Viterbi decoding. In a short while, I believe these areas will be solely the province of manufacturing engineers. Work still does remain on codes for nonbinary-input channels, such as the dual-3 code [31] useful on multiple-frequency-shift channels, and on practical implementations of high-rate codes, but these are largely development tasks. Imaginative work is still necessary to provide improvements on bursty channels beyond that given by interleaving, although it appears that interleaving does provide a good practical solution on most bursty channels (I here consider diffuse codes to be interleaved threshold decodable codes with advantage taken of the interleaving to somewhat shorten the codes). Further investigation of two-way channels is warranted, although I tend to agree with Lucky [32] that applications are quite limited.

What then are the promising areas for research and development leading to future applications? Although other shortages are currently gaining prominence, the limited availability of the useful radio spectrum and a similar spatial limitation on the number of satellites that can be placed in stationary orbit without mutual interference requires investigation from many directions. The use of Viterbi decoding to reduce intersymbol interference, although already productive [33]–[35], may contribute here as well as on telephone channels, particularly if integration with error correction can be achieved. Understanding of the multiple access of a limited volume with space, frequency, and time dimensions is still incomplete, particularly when various practical limitations and the need for network control, switching, and demand access are included. Further understanding and elaboration of such channel models as the broadcast channel [36] and the multiple-access channel [37] are required. Although some critics claim that coding will be eliminated to conserve spectrum and space, this may not be the final answer since guard spaces, guard times, and minimum antenna separations are themselves users of spectrum, time, and space, and yet do not fully eliminate mutual interference and hence errors. Again, however, simplicity may be the governing factor.

Switching problems, mentioned previously, deserve separate consideration. Telephone switching centers are now beginning to use time division as well as space division techniques [38]. Coaxial cables to most homes in a city provide possibilities for a wide mixture of communication services, but the switching and network problems are formidable. Similarly, electronic mail with sorting done electronically, rather than manually on hard copy, is likely in the near future in one form or another. It is certainly conceivable that information theory has significant contributions to make to these problems.

Source coding and rate-distortion theory are already moving to central positions theoretically and practically. Digital transmission of voice is becoming increasingly common. Techniques for compressing two-level pictures are being considered for facsimile. Experiments in teleconferencing using compressed digital television are in process. The potential for significant improvements in communication based on source coding are probably greater than for any other area, but again, the right blend of art, engineering, and theory presently escapes us.

Finally, considering the present concern with privacy and secrecy, and the prospect that such problems will increase significantly as communication services and data repositories grow, information theorists should consider a return full circle to problems that originally occupied Shannon [39]. The making and cracking of codes secure against varied levels of efforts could intrigue and occupy us indefinitely.

References

[1] J. M. Wozencraft, "Sequential decoding for reliable communication," in 1957 *Nat. IRE Conv. Rec.*, vol. 5, pt. 2, pp. 11–25; also, M.I.T. Res. Lab., Cambridge, Mass., Tech. Rep. 325.

[2] J. M. Wozencraft and B. Reiffen, *Sequential Decoding*. Cambridge, Mass.: M.I.T. Press; New York: Wiley, 1961.

[3] R. M. Fano, "A heuristic discussion of probabilistic decoding," *IEEE Trans. Inform. Theory*, vol. IT-9, pp. 64–74, Apr. 1963.

[4] J. L. Massey, *Threshold Decoding*. Cambridge, Mass.: M.I.T. Press, 1963.

[5] ——, "Advances in threshold decoding," in *Advances in Communication Systems*, A. V. Balakrishnan, Ed. New York: Academic Press, 1968.

[6] E. R. Berlekamp, *Algebraic Coding Theory*. New York: McGraw-Hill, 1968.

[7] A. J. Viterbi, "Error bounds for convolutional codes and an asymptotically optimum decoding algorithm," *IEEE Trans. Inform. Theory*, vol. IT-13, pp. 260–269, Apr. 1967.

[8] A. Kohlenberg and G. D. Forney, Jr., "Convolutional coding for channels with memory," *IEEE Trans. Inform. Theory*, vol. IT-14, pp. 618–626, Sept. 1968.

[9] J. A. Heller, "Short constraint length convolutional codes," Jet Propulsion Lab., California Inst. Technol., Space Programs Summary 37–54, vol. III, Oct./Nov. 1968, pp. 171–177.

[10] ——, "Improved performance of short constraint length convolutional codes," Jet Propulsion Lab., California Inst. Technol., Space Programs Summary 37–56, vol. III, Feb./Mar. 1969, pp. 83–84.

[11] D. Chase, "A class of algorithms for decoding block codes with channel measurement information," *IEEE Trans. Inform. Theory*, vol. IT-18, pp. 170–182, Jan. 1972.

[12] E. J. Weldon, Jr., "Decoding binary block codes on *Q*-ary output channels," *IEEE Trans. Inform. Theory*, vol. IT-17, pp. 713–718, Nov. 1971.

[13] E. R. Cacciamani, Jr., "The Spade system as applied to data communications and small earth terminal operation," *Comsat Tech. Rev.*, vol. 1, pp. 171–182, Fall 1971.

[14] D. R. Lumb, "Test and preliminary flight results on the sequential decoding of convolutionally encoded data from Pioneer IX," in *IEEE Int. Communications Conf. Rec.*, Boulder, Colo., 1969, pp. 39/1–39/8.

[15] J. P. Odenwalder, "Optimal decoding of convolutional codes," Ph.D. dissertation, School Eng. and Appl. Sci. Univ. California, Los Angeles, Mar. 1972.

[16] L. B. Hoffman and J. P. Odenwalder, "Hybrid and concatenated coding application," in *Proc. Int. Telemetering Conf.*, Los Angeles, Calif., vol. VIII, Oct. 1972.

[17] "Hybrid coding system study," LINKABIT Corp., San Diego, Calif., Final Rep. under Contract NAS2-6722; also, NASA Ames Res. Center, Moffett Field, Calif., NASA Rep. CR114486, Sept. 1972.

[18] A. J. Viterbi, "Convolutional codes and their performance in communication systems," *IEEE Trans. Commun. Technol.*, vol. COM-19, pp. 751–772, Oct. 1971.

[19] G. D. Forney, Jr., "The Viterbi algorithm," *Proc. IEEE* (Invited Paper), vol. 61, pp. 268–278, Mar. 1973.

[20] ——, "Review of random tree codes," NASA Ames Res. Cen., Moffett Field, Calif., Contract NAS2-2627, NASA CR 73176, Final Rep., Dec. 1967, appendix A.

[21] ——, "Convolutional codes II: Maximum likelihood decoding," Stanford Electron. Labs., Stanford, Calif., Tech. Rep. 7004–1, June 1972.

[22] J. A. Heller and I. M. Jacobs, "Viterbi decoding for satellite and space communication," *IEEE Trans. Commun. Technol.*, vol. COM-19, pp. 835–847, Oct. 1971.

[23] J. M. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering*. New York: Wiley, 1965, ch. 4.

[24] J. E. Savage, "Sequential decoding—the computation problem," *Bell Syst. Tech. J.*, vol. 45, pp. 149–176, Jan. 1966.

[25] I. M. Jacobs and E. R. Berlekamp, "A lower bound to the distribution of computation for sequential decoding," *IEEE Trans. Inform. Theory*, vol. IT-13, pp. 167–174, Apr. 1967.

[26] G. D. Forney, Jr., and R. M. Langelier, "A high-speed sequential decoder for satellite communications," in *Conf. Rec.*, 1969 *IEEE Int. Conf. Communications*, Boulder, Colo., June 1968, pp. 39/9–17.

[27] G. D. Forney, Jr., and E. K. Bower, "A high speed sequential decoder: Prototype design and test," *IEEE Trans. Commun. Technol.*, vol. COM-19, pp. 821–835, Oct. 1971.

[28] K. Gilhousen (with Dr. D. R. Lumb), "A very high speed hard decision sequential decoder," presented at the 1972 IEEE Nat. Telecommunications Conf., Dec. 1972.

[29] J. W. Layland and W. A. Lushbaugh, "A flexible high-speed sequential decoder for deep space channels," *IEEE Trans. on Commun. Technol.*, vol. COM-19, pp. 813–820, Oct. 1971.

[30] I. M. Jacobs, "Sequential decoding for efficient communication from deep space," *IEEE Trans. Commun. Technol.*, vol. COM-15, pp. 492–501, Aug. 1967.

[31] A. J. Viterbi, "Two constructive classes of convolutional codes for multiple signal channels" (Abstracts of Papers), in 1972 *IEEE Int. Symp. Information Theory*, p. 44.

[32] R. W. Lucky, "A survey of the communication theory literature: 1968–1973," *IEEE Trans. Inform. Theory*, vol. IT-19, pp. 725–739, Nov. 1973.

[33] G. D. Forney, Jr., "Maximum-likelihood sequence estimation of digital sequences in the presence of intersymbol interference," *IEEE Trans. Inform. Theory*, vol. IT-18, pp. 363–378, May 1972.

[34] J. K. Omura, "On optimum receivers for channels with intersymbol interference" (abstract), presented at the IEEE Int. Symp. Information Theory, Noordwijk, The Netherlands, June 1970.

[35] H. Kobayashi, "Correlative level coding and maximum-likelihood decoding," *IEEE Trans. Inform. Theory*, vol. IT-17, pp. 586–594, Sept. 1971.

[36] T. M. Cover, "Broadcast channels," *IEEE Trans. Inform. Theory*, vol. IT-18, pp. 2–14, Jan. 1972.

[37] H. H. J. Liao, "Multiple access channels," Ph.D. dissertation, Univ. of Hawaii, Honolulu; also, The Aloha System, Tech. Rep. A72-2, Sept. 1972.

[38] G. D. Johnson, "No. 4 ESS—Long distance switching for the future," *Bell. Lab. Rec.*, vol. 51, pp. 226–232, Sept. 1973.

[39] C. E. Shannon, "Communication theory of secrecy systems," *Bell Syst. Tech. J.*, vol. 28, pp. 656–715, Oct. 1949.