

Session 2

Complex Numbers

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Complex Number Review (1)

- A complex number, c , can be defined as an ordered pair

$$c = (a, b)$$

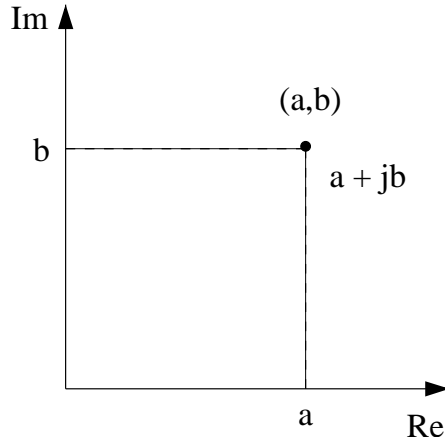
of real numbers a and b , where a and b are known as the real and imaginary parts of c .

- Since the complex number defined above consists of two components, it is typically plotted as a single point on a two dimensional plane. Defining the complex number as an ordered pair is referred to as a rectangular coordinate representation.
- Engineers generally do not use the ordered pair representation, instead they represent the complex number c as

$$c = a + jb.$$

Complex Number Review (2)

- A graphical representation of a complex number in rectangular coordinates is shown in the following graph:



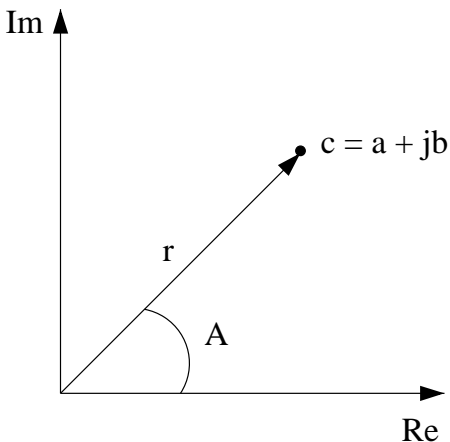
Complex Number Review (3)

- A complex number can also be represented in polar form using

$$a = r \cos A, \quad (1)$$

$$b = r \sin A, \quad (2)$$

with r and A defined in this figure:



Complex Number Review (4)

- The magnitude, r , and angle, A , can be determined using the relationships

$$r = |c| = \sqrt{a^2 + b^2},$$
$$A = \tan^{-1} \left(\frac{b}{a} \right)$$

which are determined from the previous figure.

- The polar form of c is represented as $r \angle A$.
- Note that \tan^{-1} is quadrant dependent and thus in MATLAB `angle` or `atan2` should be used. See the section titled, What is a Polar Angle?, on page 108 in the book.

Complex Number Review (5)

- The third representation of a complex number is the exponential form, which can be derived by substituting

$$a = r \cos A, \quad b = r \sin A,$$

into

$$c = a + jb.$$

and using Euler's formula

$$e^{jA} = \cos A + j \sin A$$

to give

$$\begin{aligned} c &= r(\cos A + j \sin A) \\ &= r e^{jA} \end{aligned}$$

- In the engineering field this geometrically represents a phasor.

MATLAB Functions for Complex Numbers

- `conj(c)`: Complex conjugate of c . If $c=a+jb$, then $\text{conj}(c)=a-jb$.
- `real(c)`: Determines the real part of c .
- `imag(c)`: Determines the imaginary part of c .
- `isreal(c)`: Returns true (1) if array c has no imaginary elements.
- `abs(c)`: Determines the magnitude of c .
- `angle(c)`: Determines the angle of c .
- `complex(a,b)`: Converts real components a and b into a complex number c .